BATCH PROOFS ARE STATISTICALLY HIDING

CHETHAN KAMATH




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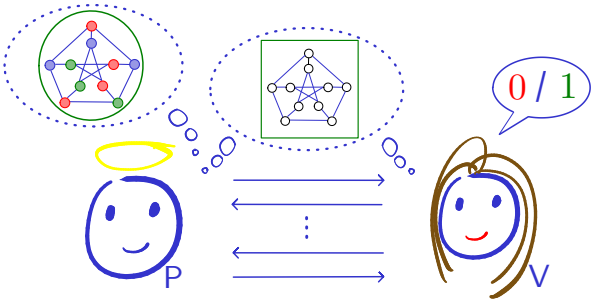
TEL AVIV UNIVERSITY אוניברסיטת תל אביב

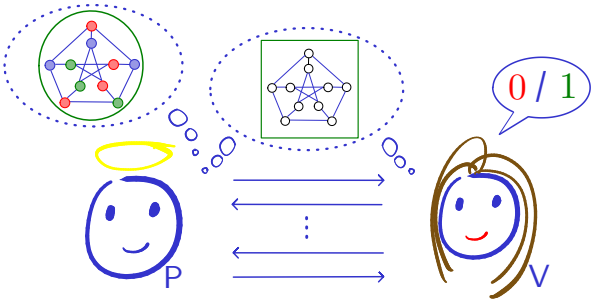
NUS National University of Singapore

TECHNION Israel Institute of Technology



INTERACTIVE PROTOCOLS

COMPLETENESS ◆ ◆◆ SOUNDNESS



INTERACTIVE PROTOCOLS

COMPLETENESS

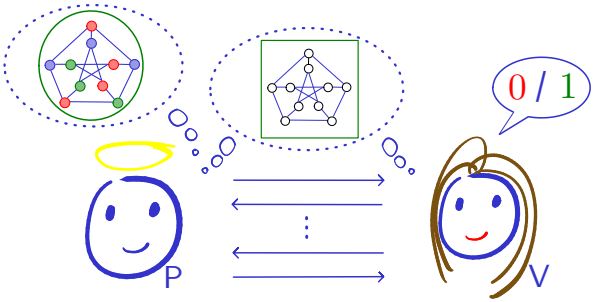
SOUNDNESS

SUCCINCTNESS

BATCHING

HIDING

WITNESS IND.



INTERACTIVE PROTOCOLS

COMPLETENESS ◆ ◆ SOUNDNESS

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BATCHING WITNESS IND.

PLAN

- BACKGROUND
  - DEFINITIONS: BATCH PROOFS STAT. WITNESS INDISTINGUISHABILITY (SWI)
  - WHAT CAN BE BATCHED?

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INTERACTIVE PROTOCOL FOR $\mathcal{L}$

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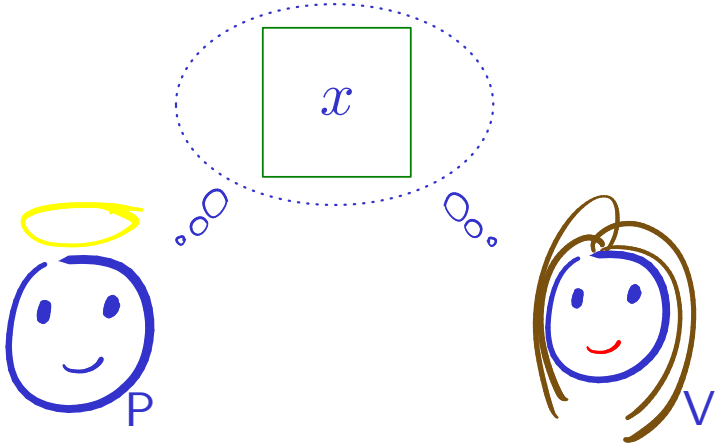

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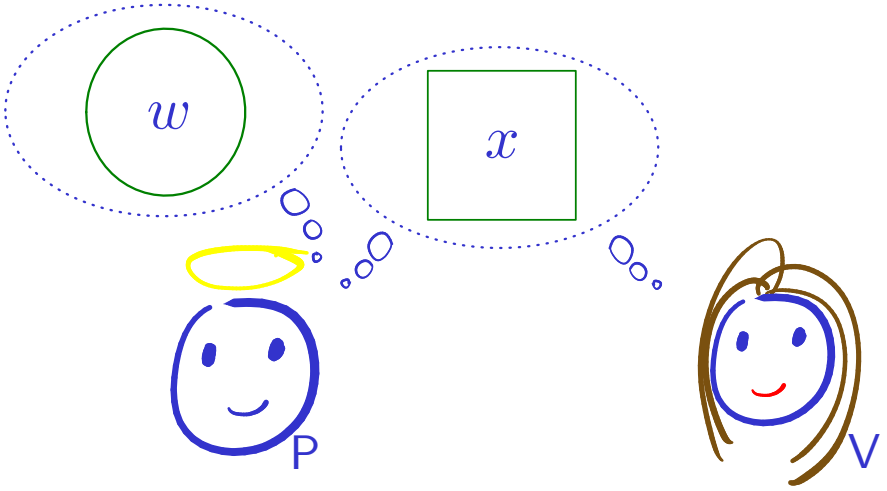
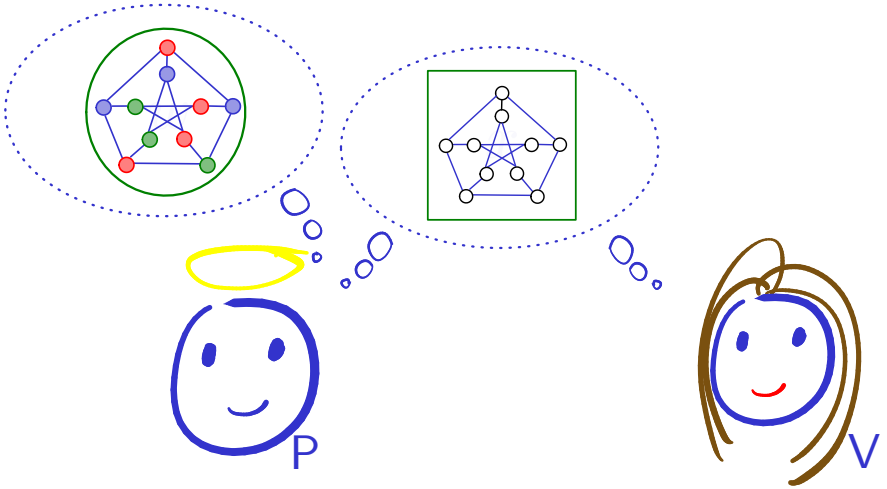


INTERACTIVE PROTOCOL FOR $\mathcal{L}$ (E.G.: 3-COLORING)



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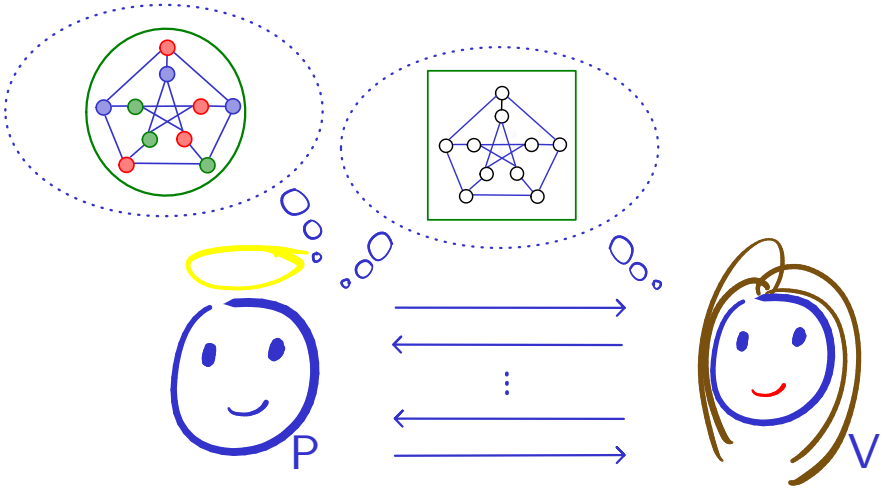

(PUBLIC-COIN)

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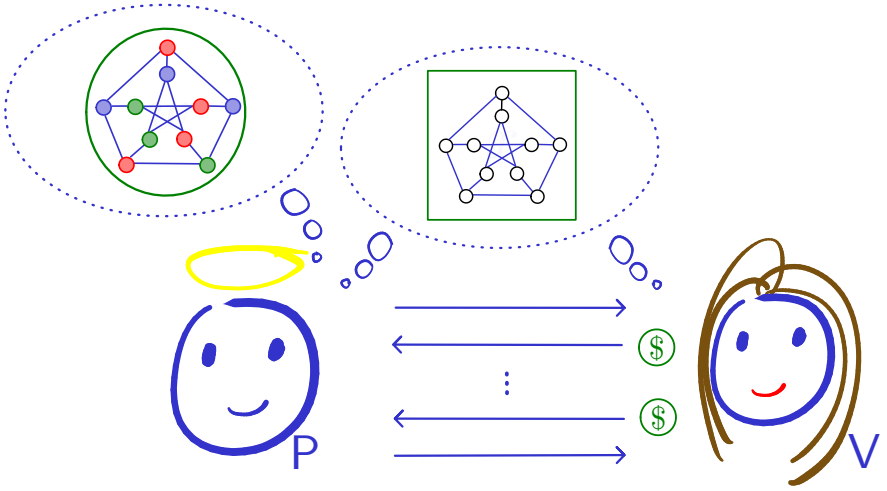

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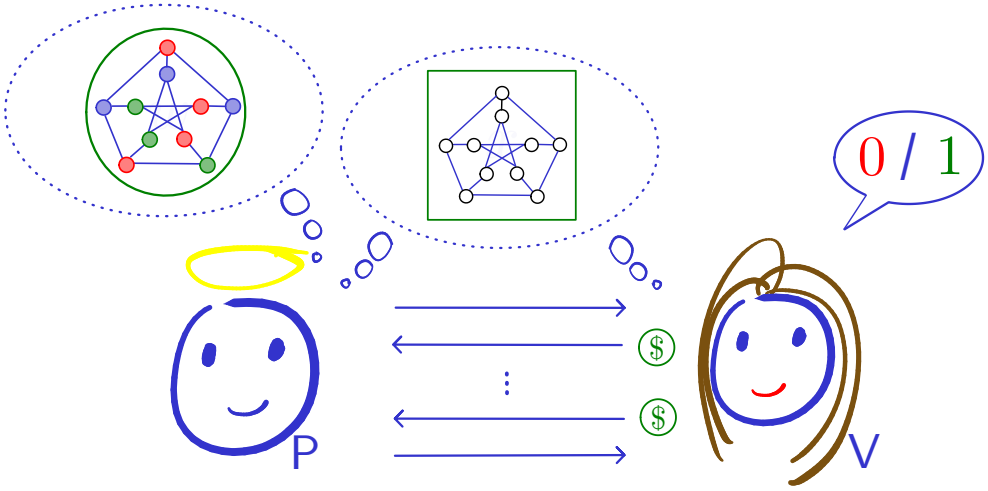

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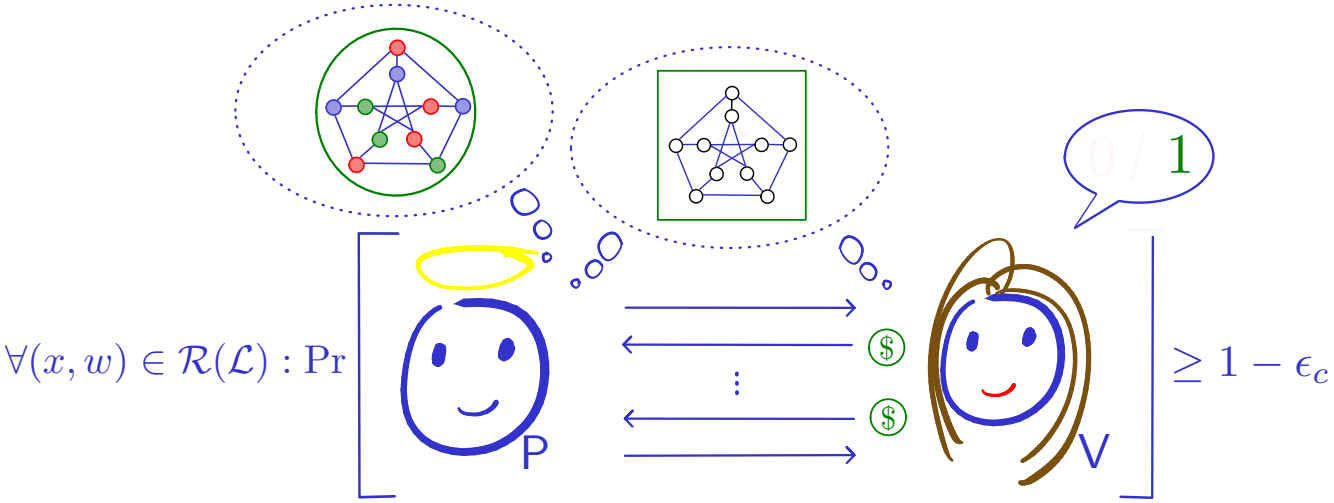


$$\forall (x, w) \in \mathcal{R}(\mathcal{L}) : \Pr[\ \langle P, V \rangle(x) = 1\ ] \geq 1 - \epsilon_c$$

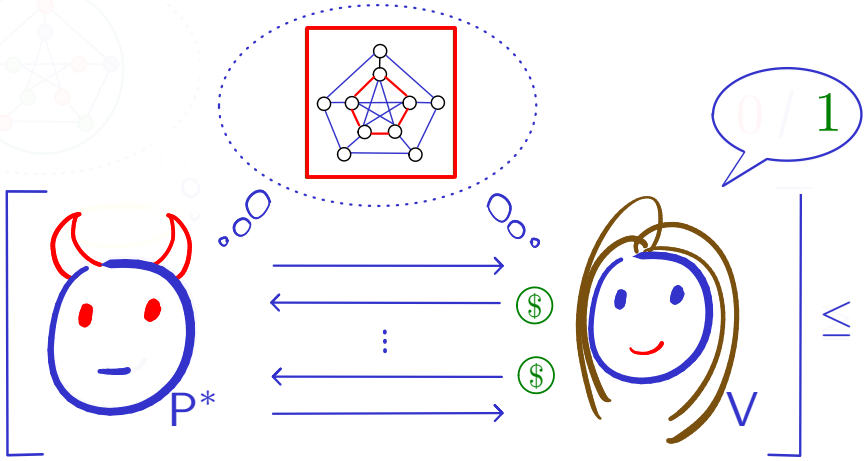
◆ REQUIREMENTS

◆ COMPLETENESS

HONEST P CONVINCES V OF TRUE STATEMENTS

(PUBLIC-COIN)

INTERACTIVE PROTOCOL FOR $\mathcal{L}$ (E.G.: 3-COLORING)



$$\forall x \notin \mathcal{L} : \Pr\left[\text{P}^* \leftrightarrows \text{V}\right] \leq \epsilon_s$$

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SOUNDNESS

MALICIOUS P* CANNOT CONVINCE V OF FALSE STATEMENTS

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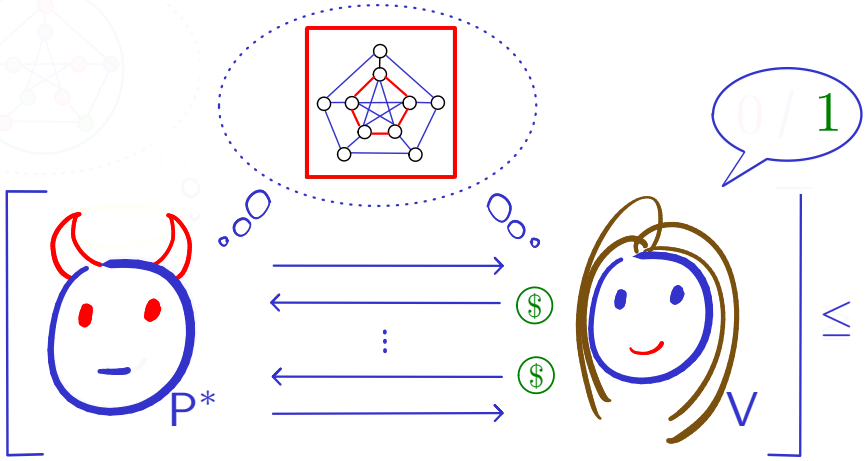

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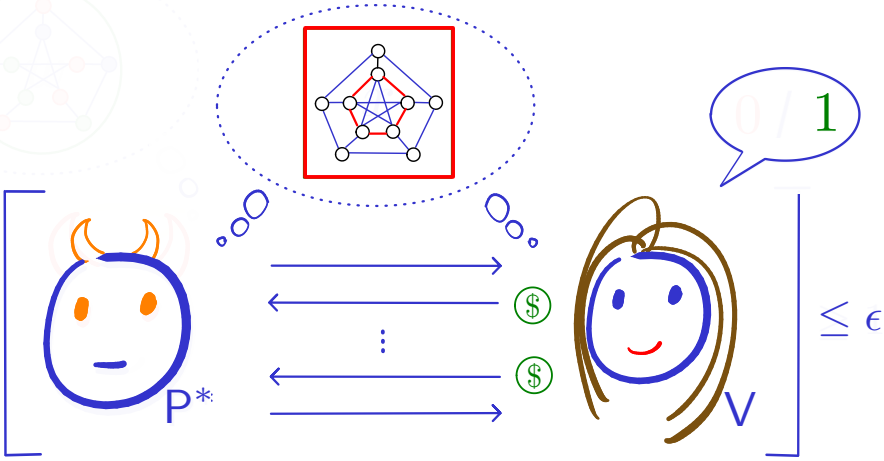
UNBOUNDED
STATISTICAL SOUNDNESS = PROOF

(PUBLIC-COIN)

INTERACTIVE PROTOCOL FOR $\mathcal{L}$ (E.G.: 3-COLORING)



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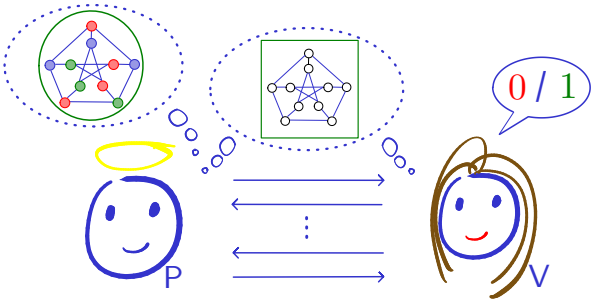
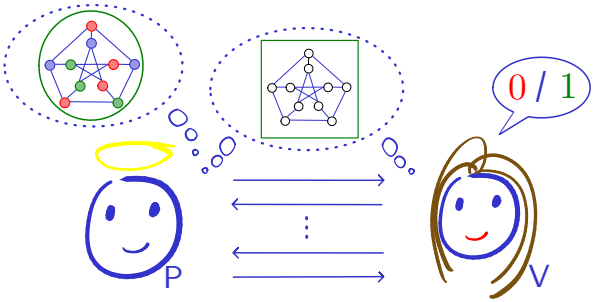
- UNBOUNDED
  STATISTICAL SOUNDNESS = PROOF
- BOUNDED
  COMPUTATIONAL SOUNDNESS=ARGUMENT

...



INTERACTIVE PROTOCOLS

- COMPLETENESS
- SOUNDNESS



INTERACTIVE PROTOCOLS

- COMPLETENESS
- SOUNDNESS
- SUCCINCTNESS
  "CAN WE REDUCE COMMUNICATION?"
- HIDING
  "CAN WE HIDE THE WITNESS?"

BATCH PROTOCOL FOR $\mathcal{L}^t$

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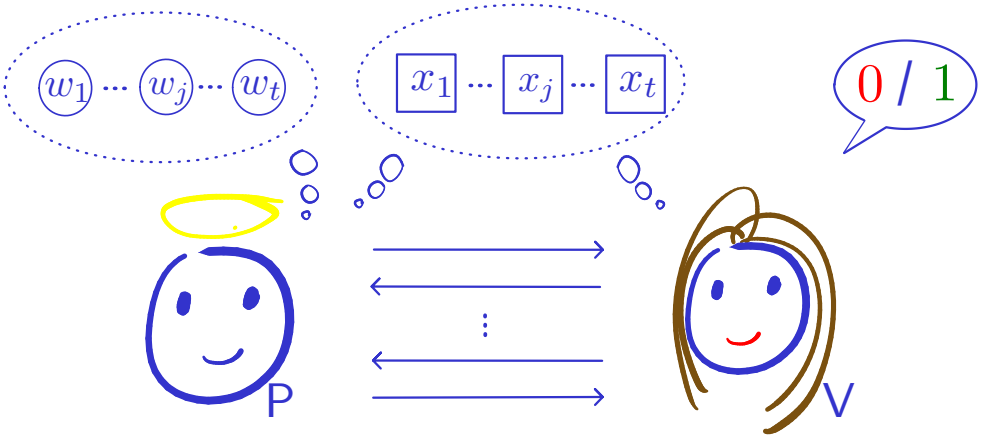

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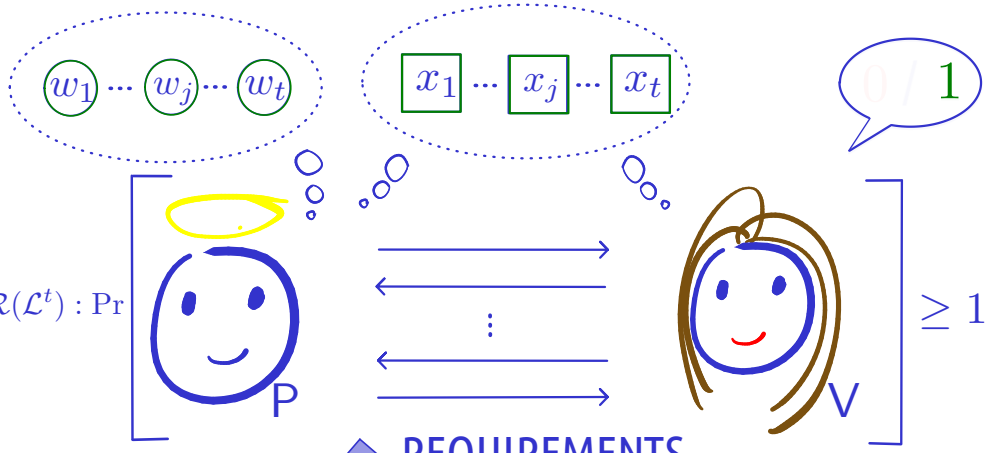

$\forall (x_1, w_1), \cdots, (x_t, w_t) \in \mathcal{R}(\mathcal{L}^t) : \Pr[\ \ ] \geq 1 - \epsilon_c$

REQUIREMENTS

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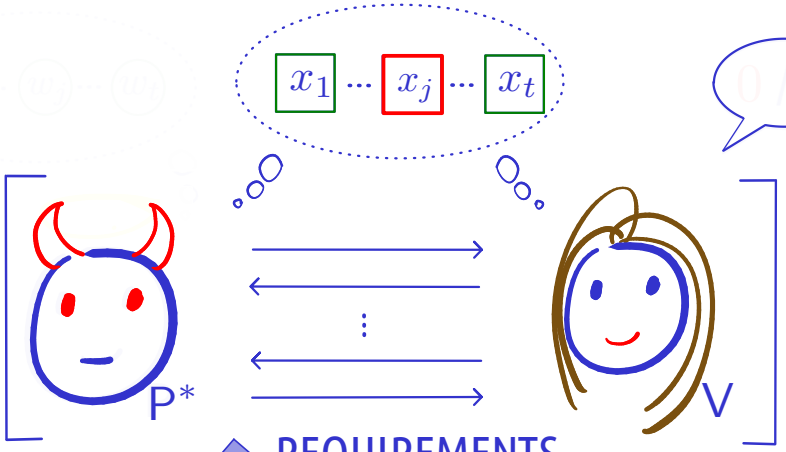
$\forall x_1, \ldots, x_t$ s.t. $\exists x_j \notin \mathcal{L} : \Pr$



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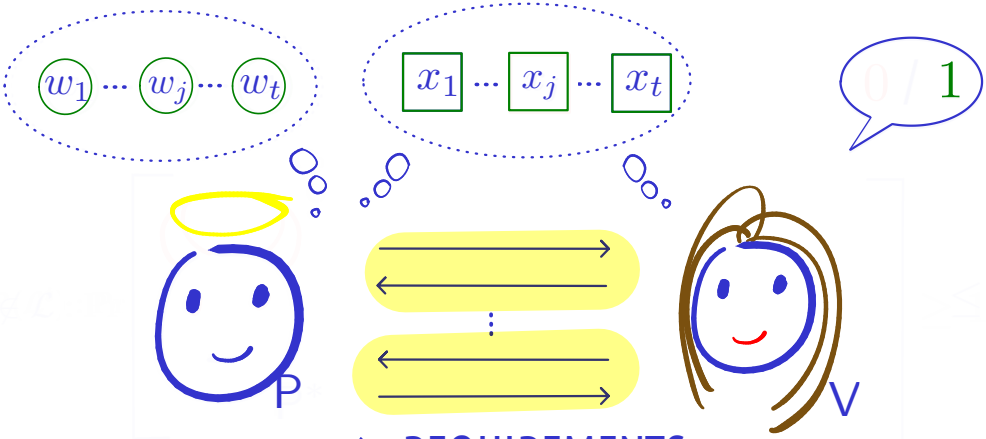

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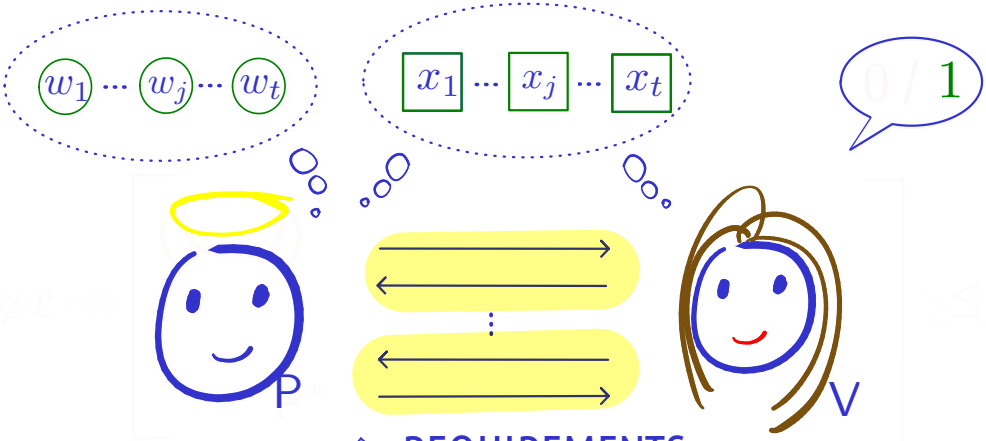

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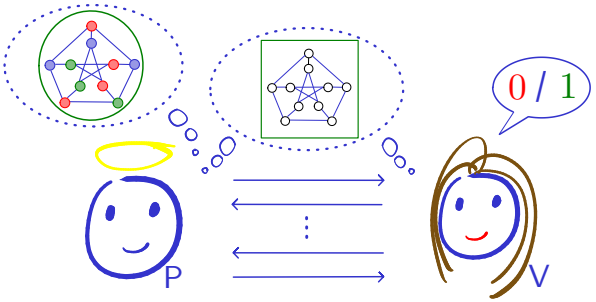
TRIVIAL IF P UNBOUNDED VIA IP=PSPACE



INTERACTIVE PROTOCOLS

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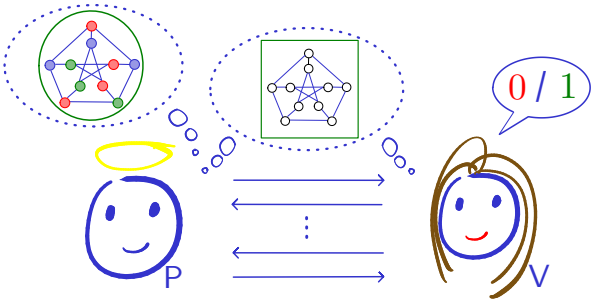

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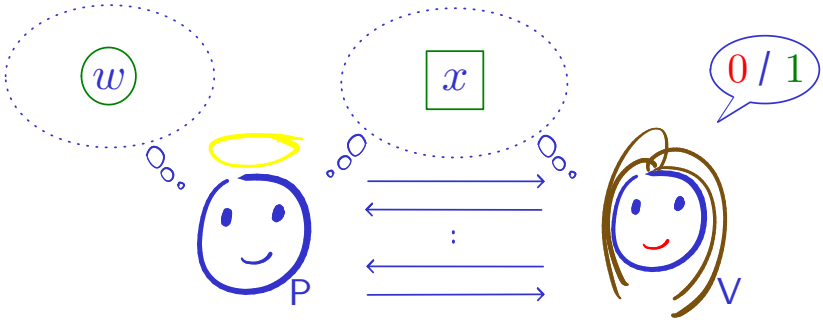
WITNESS INDISTINGUISHABILITY [FLS90]



(HONEST-VERIFIER)

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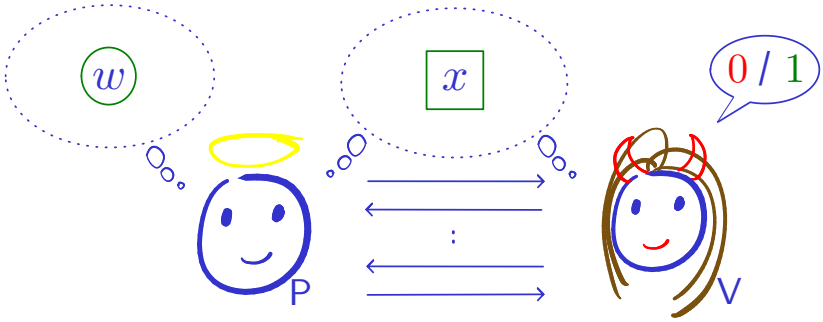

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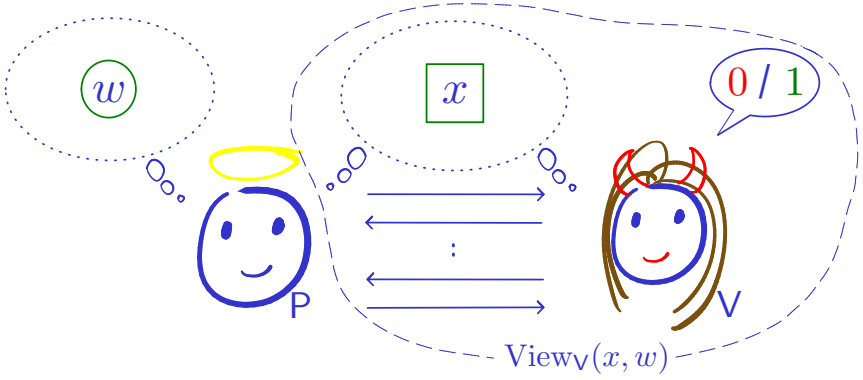

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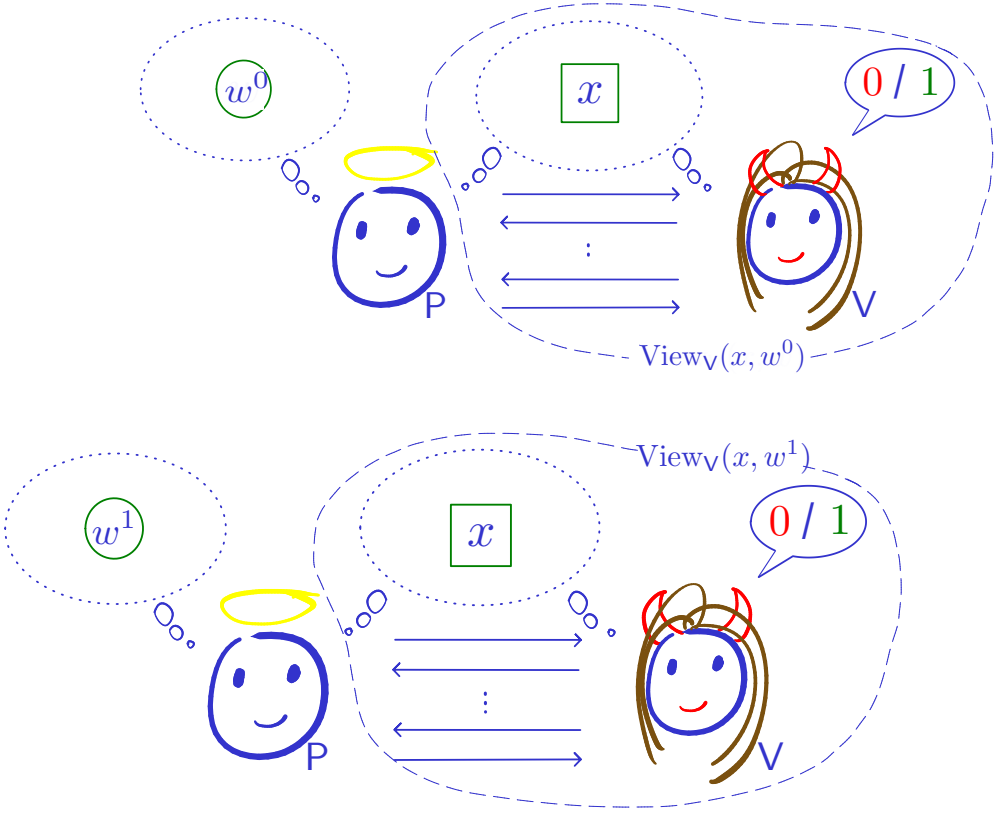

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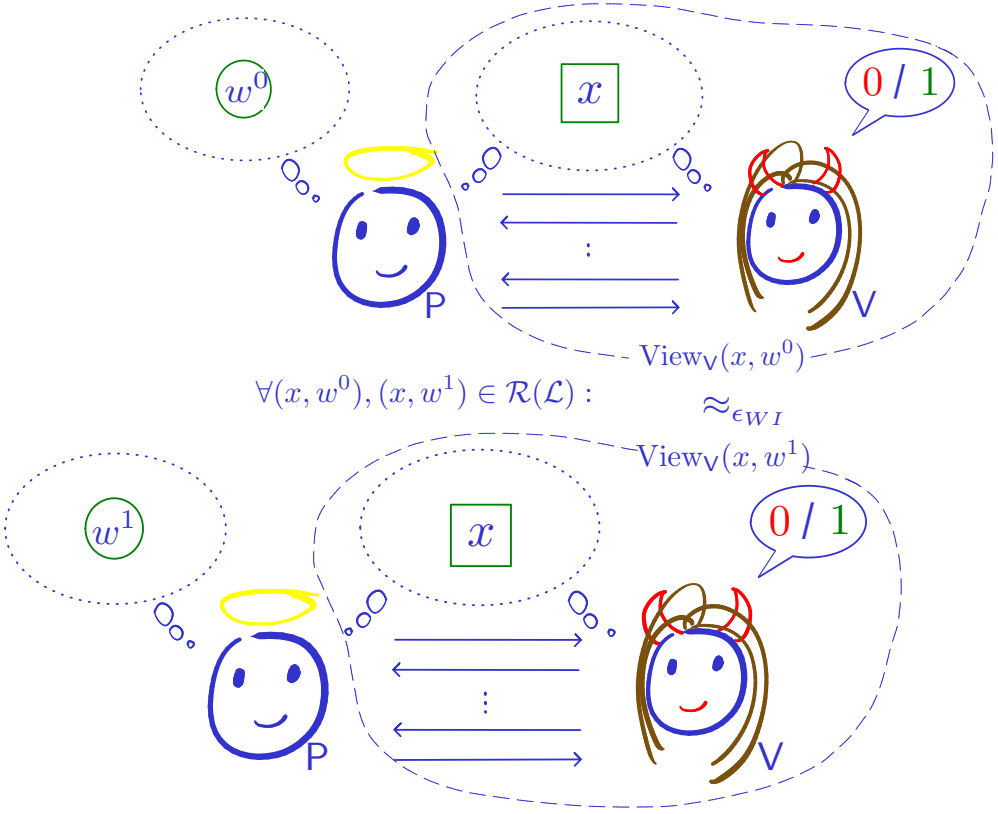
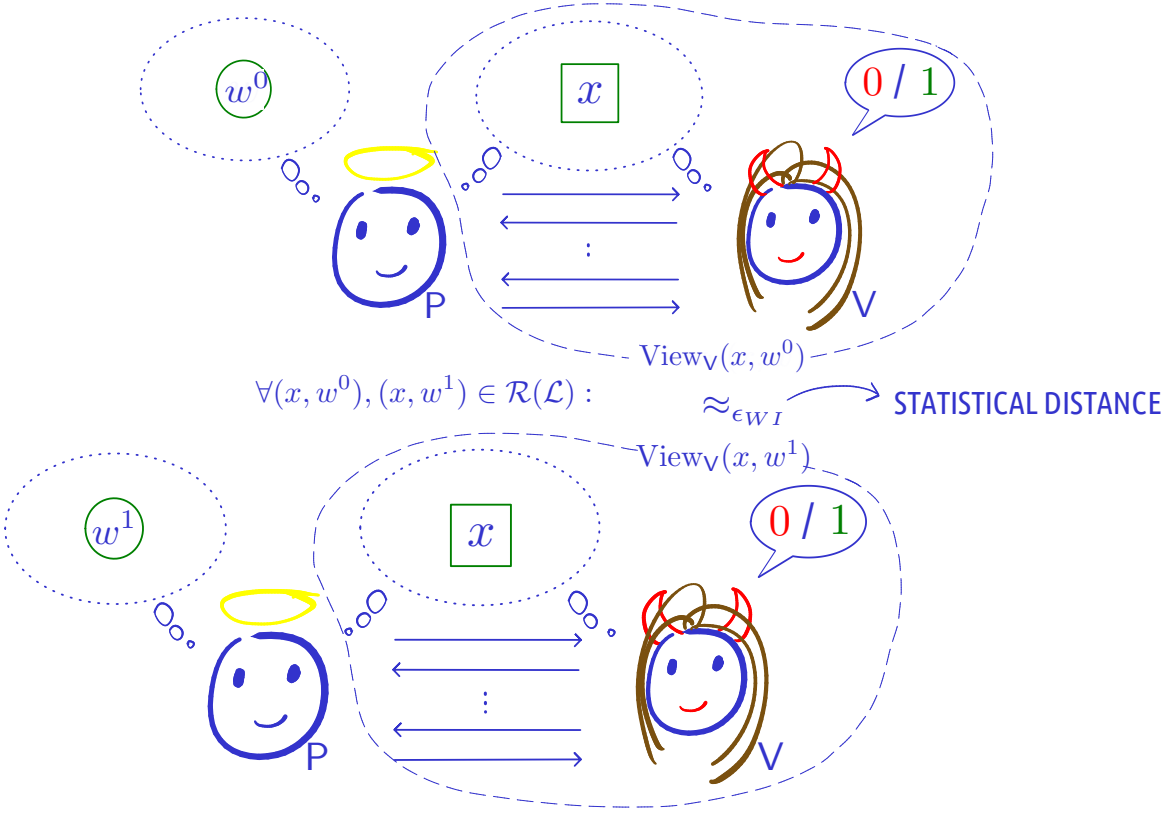


(HONEST-VERIFIER) STATISTICAL WITNESS INDISTINGUISHABILITY [FLS90]



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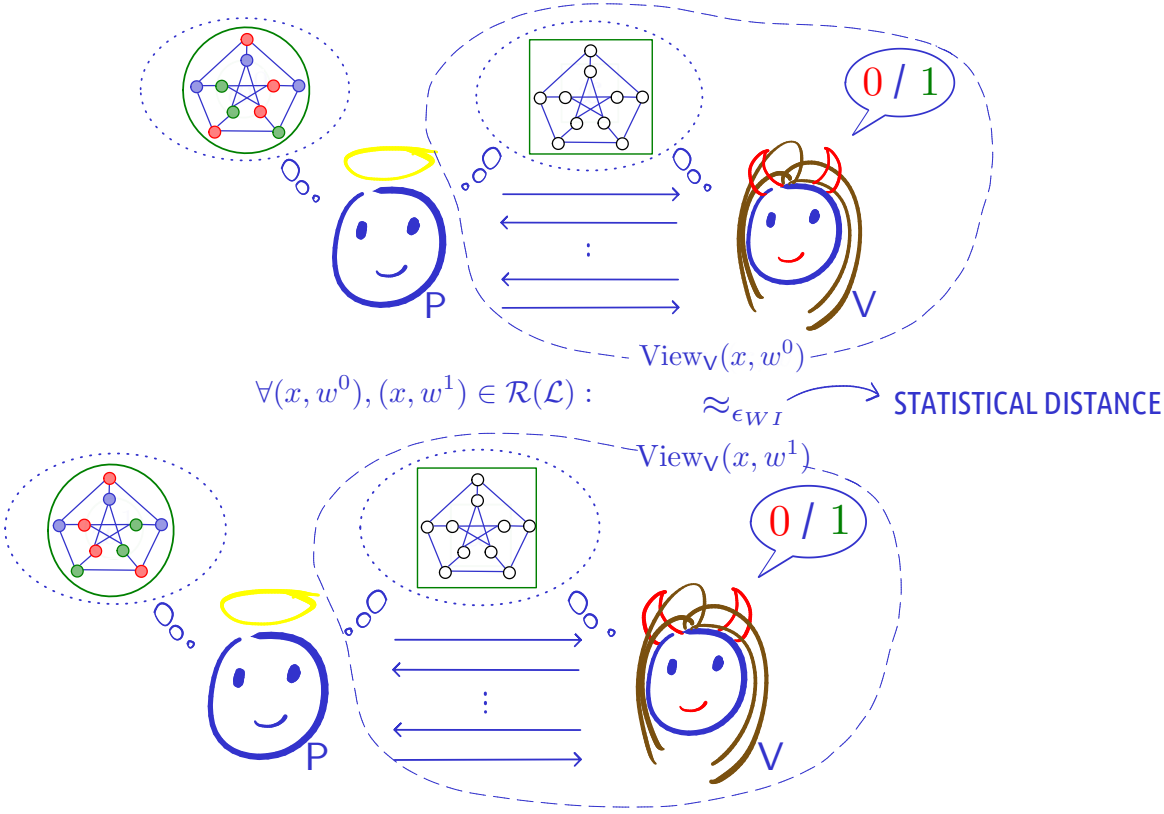

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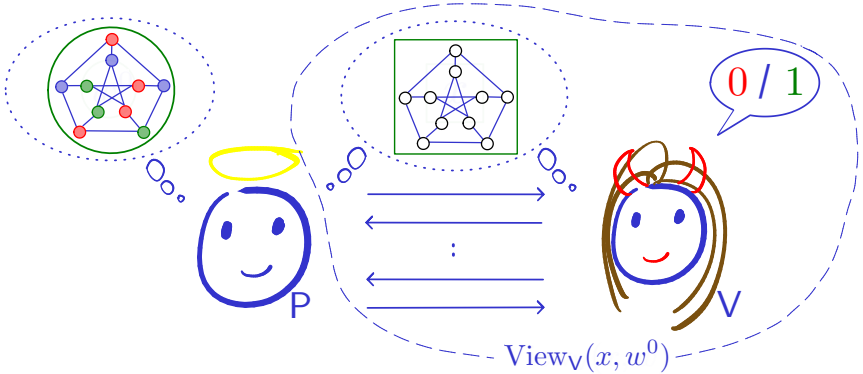
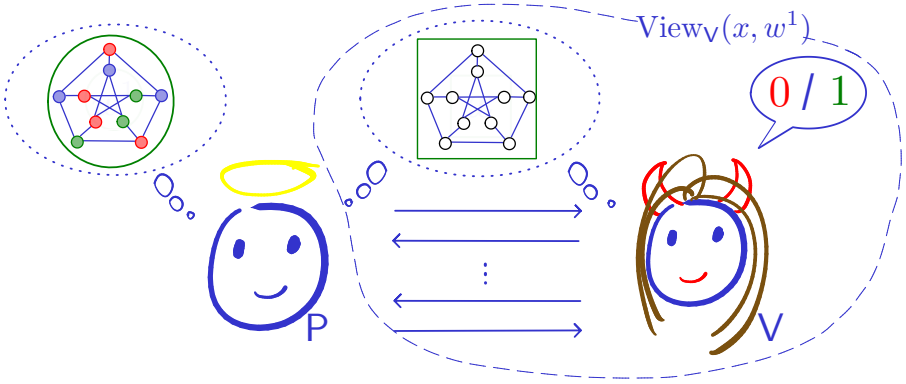


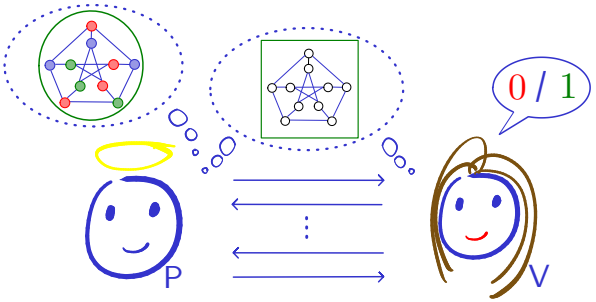
$\text{View}_V(x, w^0)$

$\forall (x, w^0), (x, w^1) \in \mathcal{R}(\mathcal{L}) :$ $\approx_{\epsilon_{WI}}$ STATISTICAL DISTANCE

$\text{View}_V(x, w^1)$



TRIVIAL IF P UNBOUNDED



INTERACTIVE PROTOCOLS

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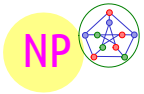
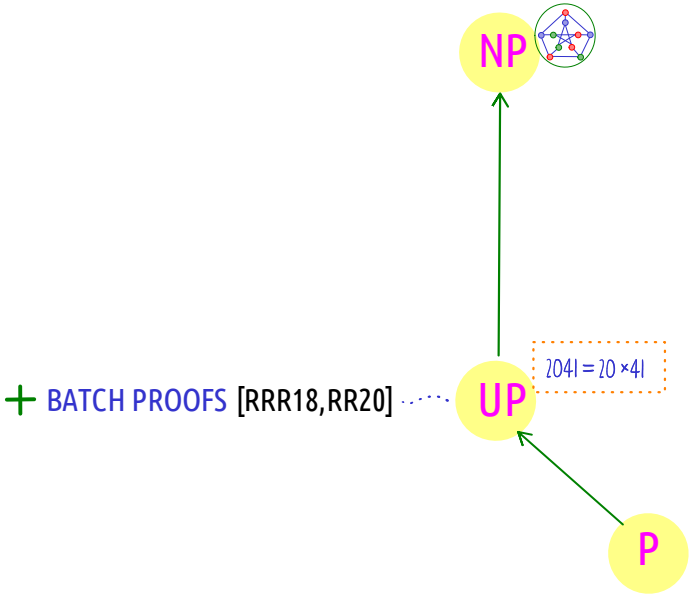
WHAT CAN BE BATCHED?



WHAT CAN BE BATCHED?

- PRIOR WORKS



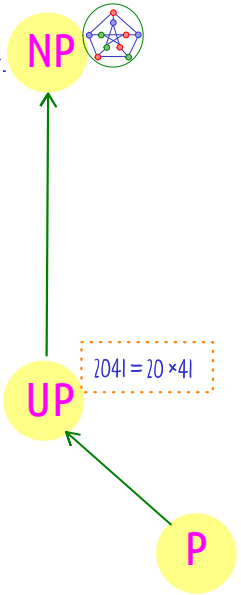
WHAT CAN BE BATCHED?

- PRIOR WORKS

? BATCH PROOFS NOT KNOWN (WHEN HONEST PROVER IS EFFICIENT)

\+ NON-INTERACTIVE BATCH ARGUMENTS FROM STANDARD ASSUMPTIONS [CJJ22, WW22, HJKS22]

\+ BATCH PROOFS [RRR18, RR20]



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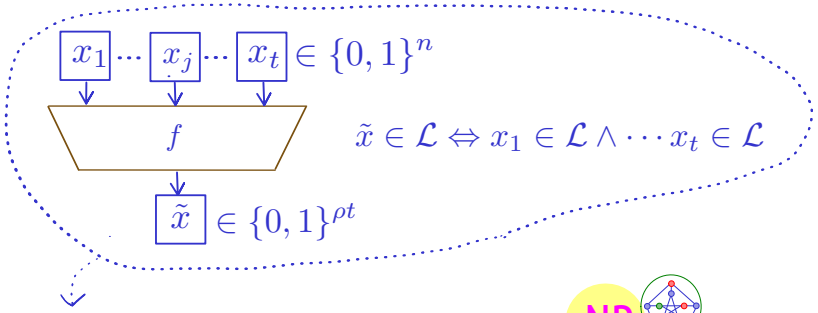

PRIOR WORKS

− INSTANCE COMPRESSION UNLIKELY [D15]

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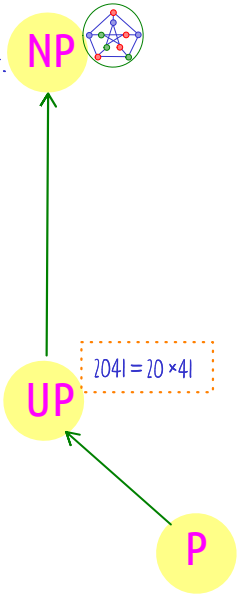
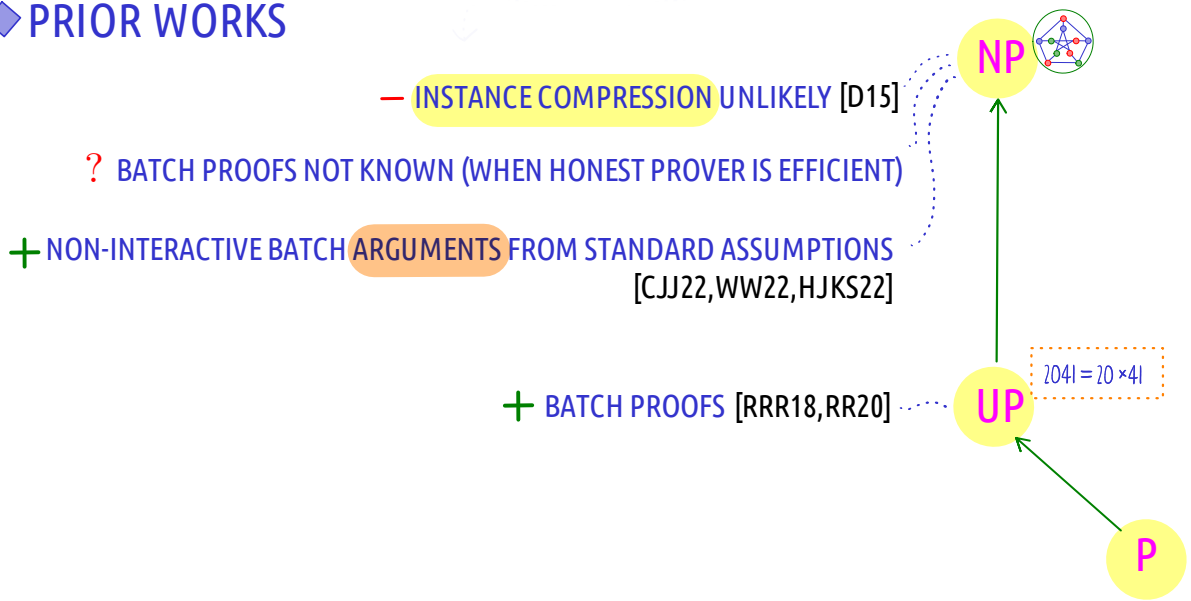


WHAT CAN BE BATCHED?

- THIS WORK: WHAT HAPPENS WHEN BATCHING IS POSSIBLE?

- PRIOR WORKS



WHAT CAN BE BATCHED?

- THIS WORK: WHAT HAPPENS WHEN BATCHING IS POSSIBLE?
  - + BATCH PROOF FOR NP IMPLIES SWI PROOF FOR NP

- PRIOR WORKS
  - − INSTANCE COMPRESSION UNLIKELY [D15]
  - ? BATCH PROOFS NOT KNOWN (WHEN HONEST PROVER IS EFFICIENT)
  - + NON-INTERACTIVE BATCH ARGUMENTS FROM STANDARD ASSUMPTIONS [CJJ22, WW22, HJKS22]
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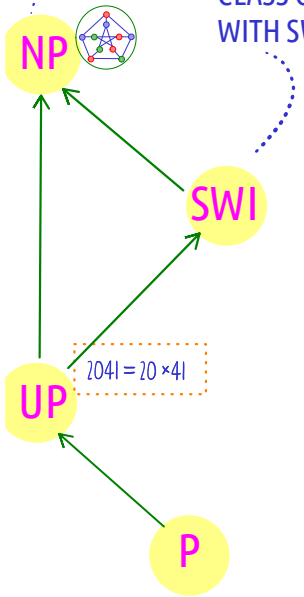

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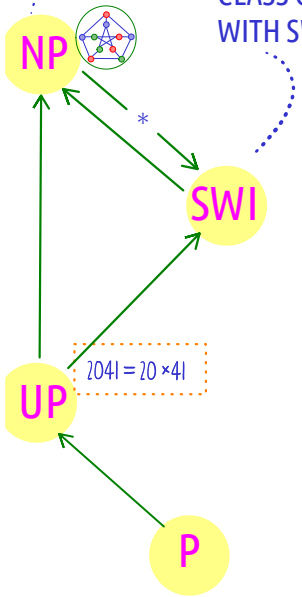

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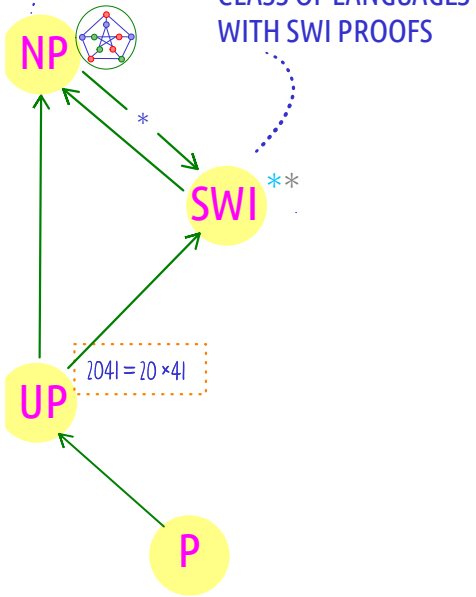

WHAT CAN BE BATCHED?

- THIS WORK: WHAT HAPPENS WHEN BATCHING IS POSSIBLE?
  - + BATCH PROOF FOR NP IMPLIES SWI PROOF** FOR NP
  - − *
  - + (NI) BATCH ARGUMENT FOR NP IMPLIES (NI) SWI ARGUMENT** FOR NP
- PRIOR WORKS
  - − INSTANCE COMPRESSION UNLIKELY [D15]
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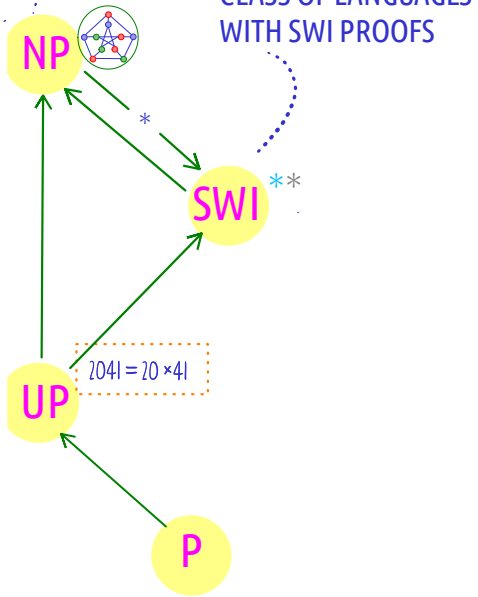

PLAN

- BACKGROUND
  - DEFINITIONS: BATCH PROOFS STAT. WITNESS INDISTINGUISHABILITY (SWI)
  - WHAT CAN BE BATCHED?
- **MAIN RESULT**
  - **BATCH PROOFS $\Rightarrow$ SWI PROOFS**
  - **TECHNICAL OVERVIEW**
- MORE RESULTS
  - ONE-WAY FUNCTION + BATCH ARGUMENTS $\Rightarrow$ SZK ARGUMENTS
  - NON-INTERACTIVE (NI) BATCH ARGUMENTS $\Rightarrow$ NI SZK ARGUMENTS
- OPEN QUESTIONS

BATCH PROOFS $\Rightarrow$ SWI PROOFS

- *MAIN THEOREM.*

(PUBLIC-COIN) BATCH PROOF FOR $\mathcal{L}^t$ WITH COMPRESSION RATE ρ

$\Downarrow$

(PUBLIC-COIN) HV-SWI PROOF** FOR $\mathcal{L}$

BATCH PROOFS $\Rightarrow$ SWI PROOFS

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- NEGLIGIBLE SOUNDNESS ERROR
- WI ERROR $\epsilon_{WI} \approx \sqrt{\rho}$: CAN BE INVERSE-POLYNOMIAL $(*)$
- PROVER IS NON-UNIFORM $(*)$

BATCH PROOFS $\Rightarrow$ SWI PROOFS

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$\Downarrow$ [GSV98]

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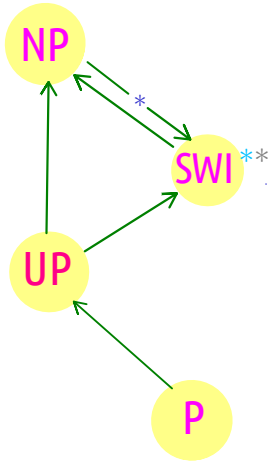
– *COROLLARY.* IF NP DOESN'T HAVE HV-SWI PROOFS**
THEN IT DOESN'T HAVE BATCH PROOFS



BATCH PROOFS $\Rightarrow$ SWI PROOFS

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  SWI PROOF** FOR $\mathcal{L}$

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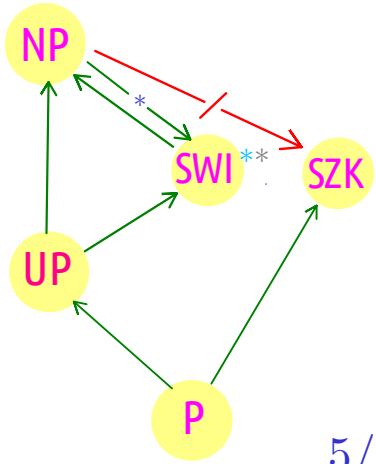
  - PROVER IS NON-UNIFORM $(*)$

- *COROLLARY.* IF NP DOESN'T HAVE HV-SWI PROOFS** THEN IT DOESN'T HAVE BATCH PROOFS
  - $\mathbf{NP} \subseteq \mathbf{SZK} \Rightarrow$ POLYNOMIAL HIERARCHY COLLAPSES [F89,AH91]
  - SWI SHOULD NOT BE MUCH MORE POWERFUL THAN SZK



BATCH PROTOCOL $\mapsto$ SWI PROTOCOL

$\Pi = (\mathsf{P}, \mathsf{V})$ $\quad$ $\Pi' = (\mathsf{P}', \mathsf{V}')$

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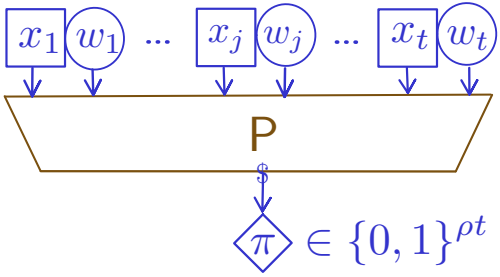

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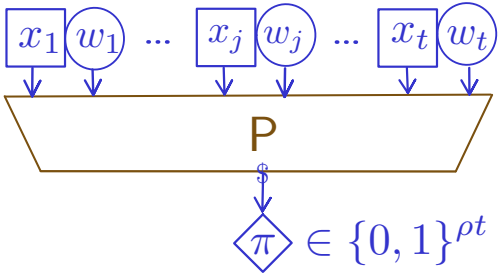

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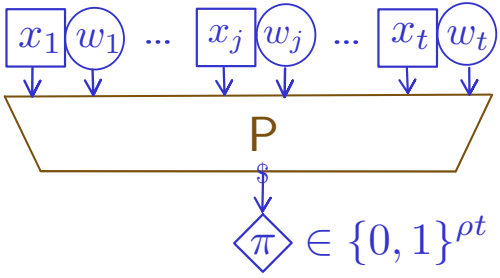

INTUITION

$\blacklozenge \mapsto \blacklozenge$

P IS COMPRESSING $\Rightarrow$ LOSES INFORMATION ABOUT INPUT $\mapsto$ SWI OF Π'

BATCH PROTOCOL $\Pi = (\mathsf{P}, \mathsf{V})$ $\mapsto$ SWI PROTOCOL $\Pi' = (\mathsf{P}', \mathsf{V}')$

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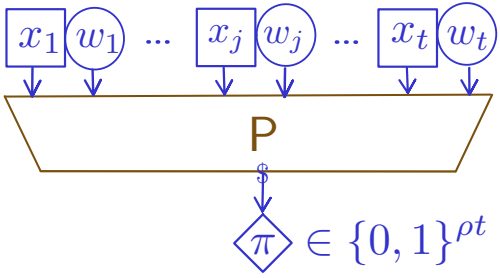

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Π COMPLETE AND SOUND $\Rightarrow$ Π' COMPLETE AND SOUND

BATCH PROTOCOL $\mapsto$ SWI PROTOCOL...

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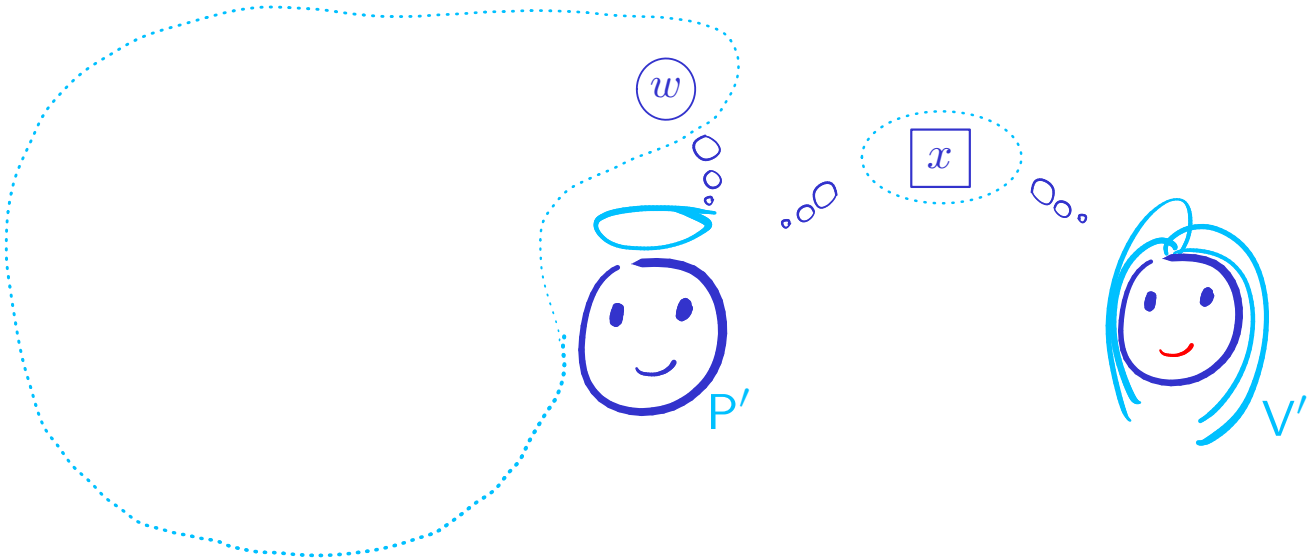

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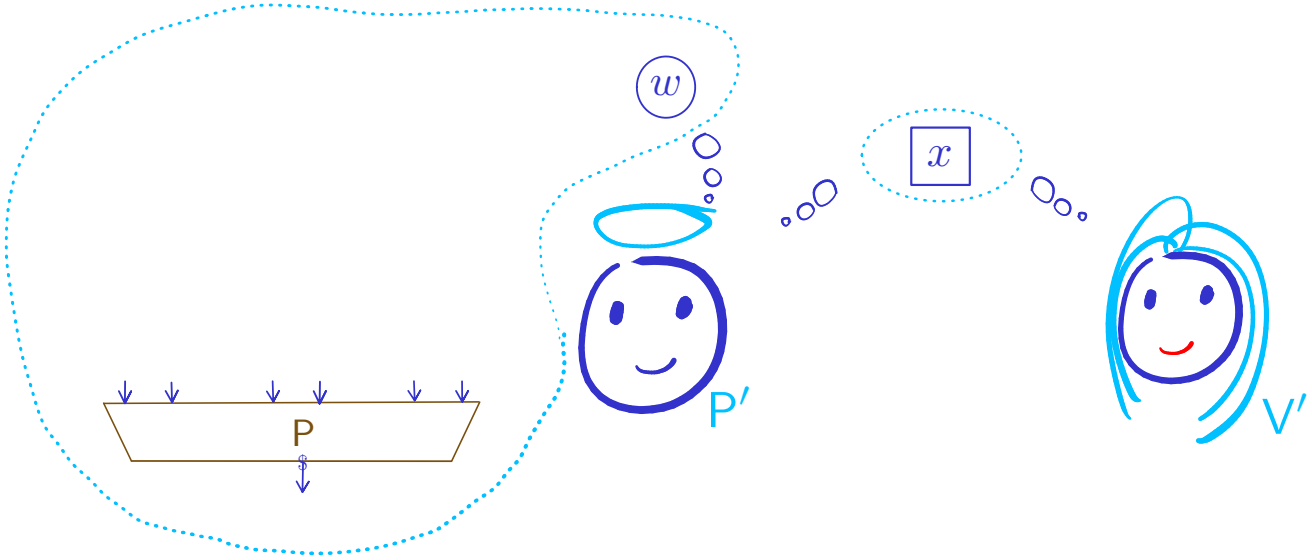

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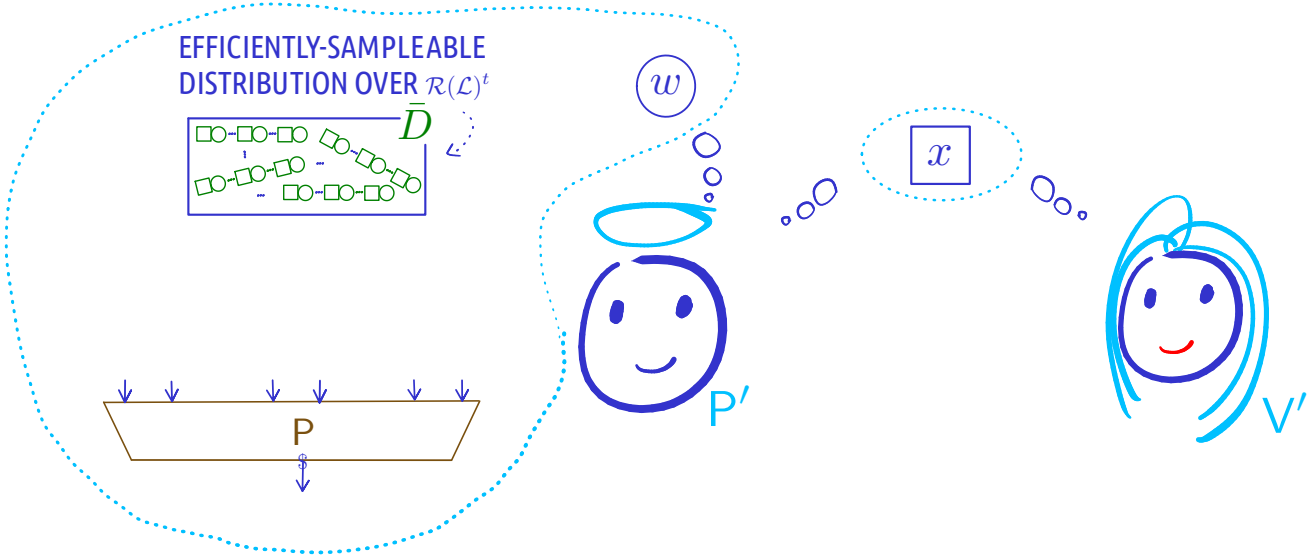

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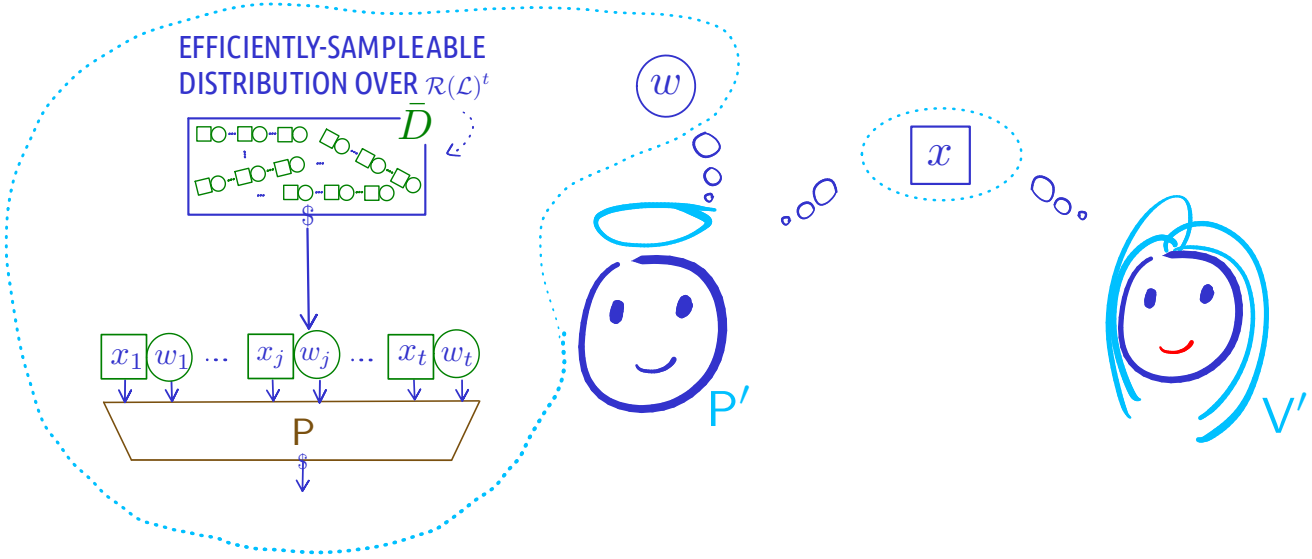

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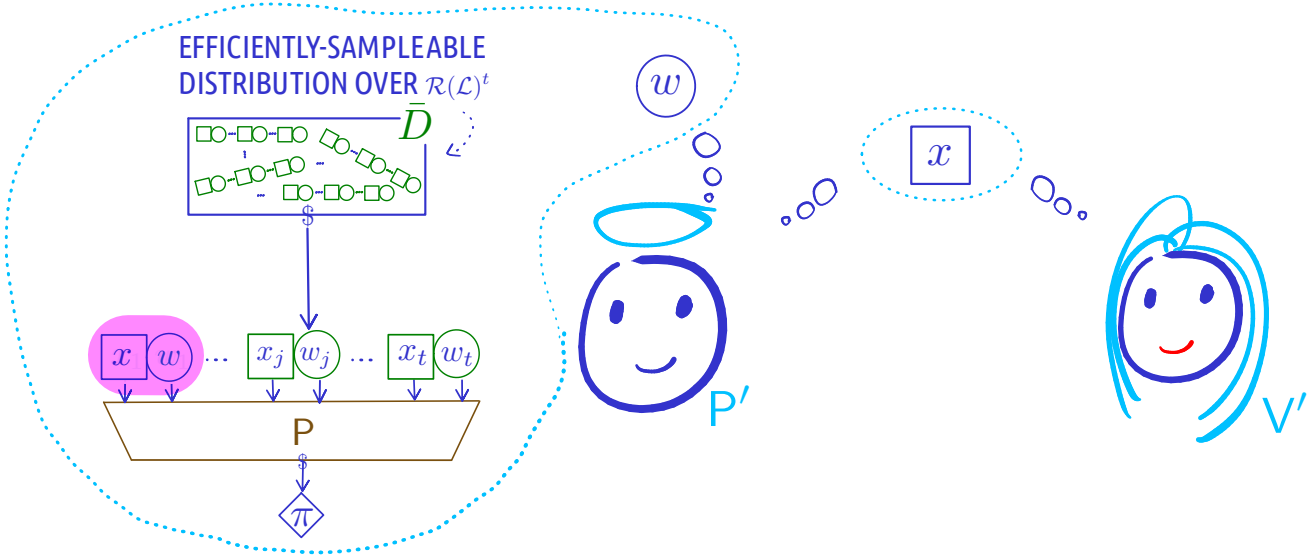

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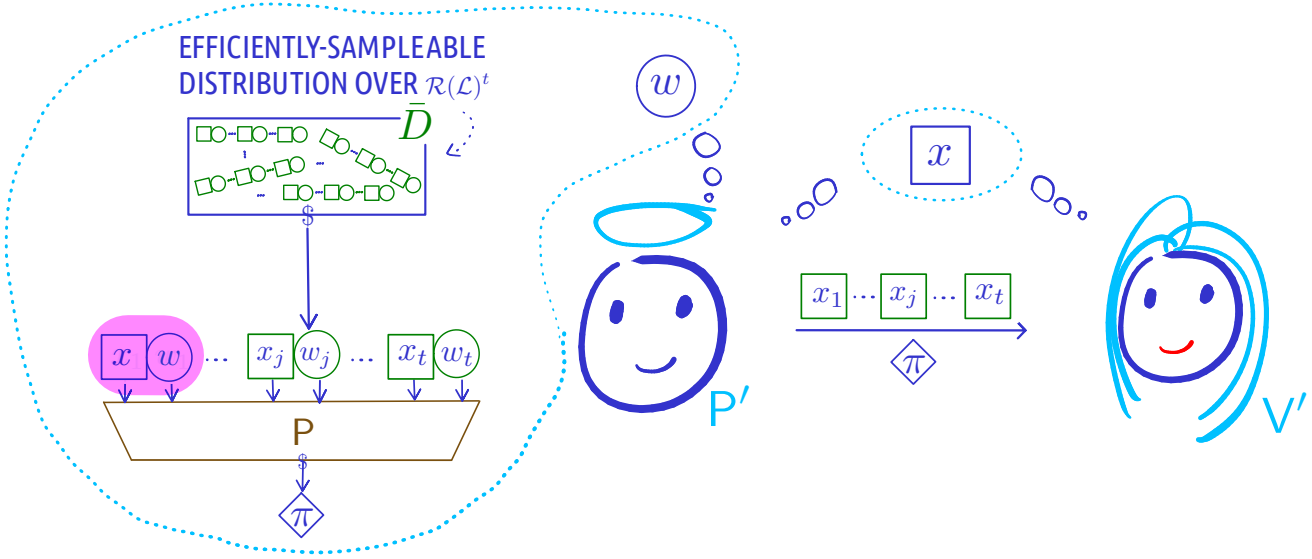

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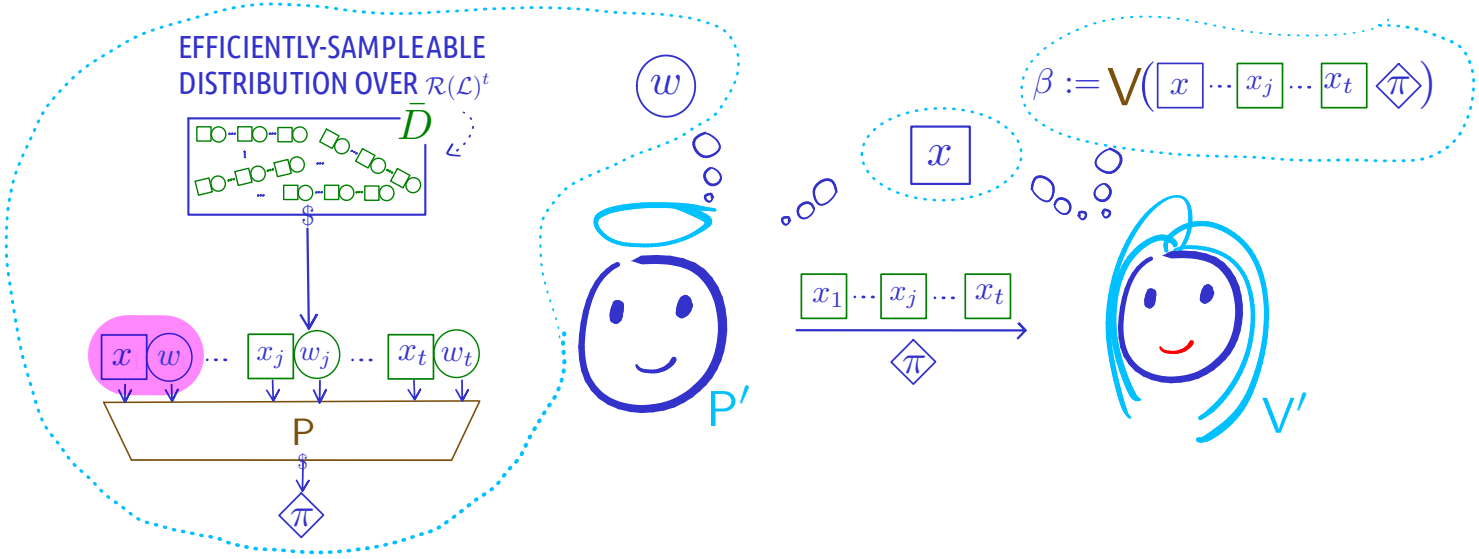

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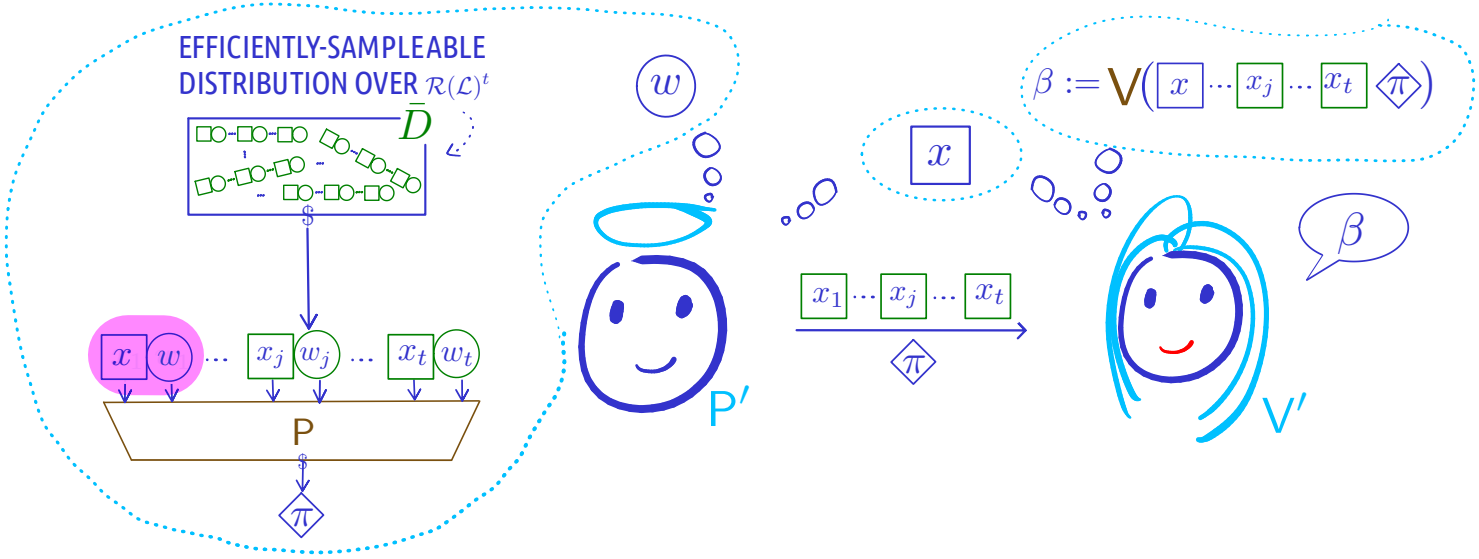

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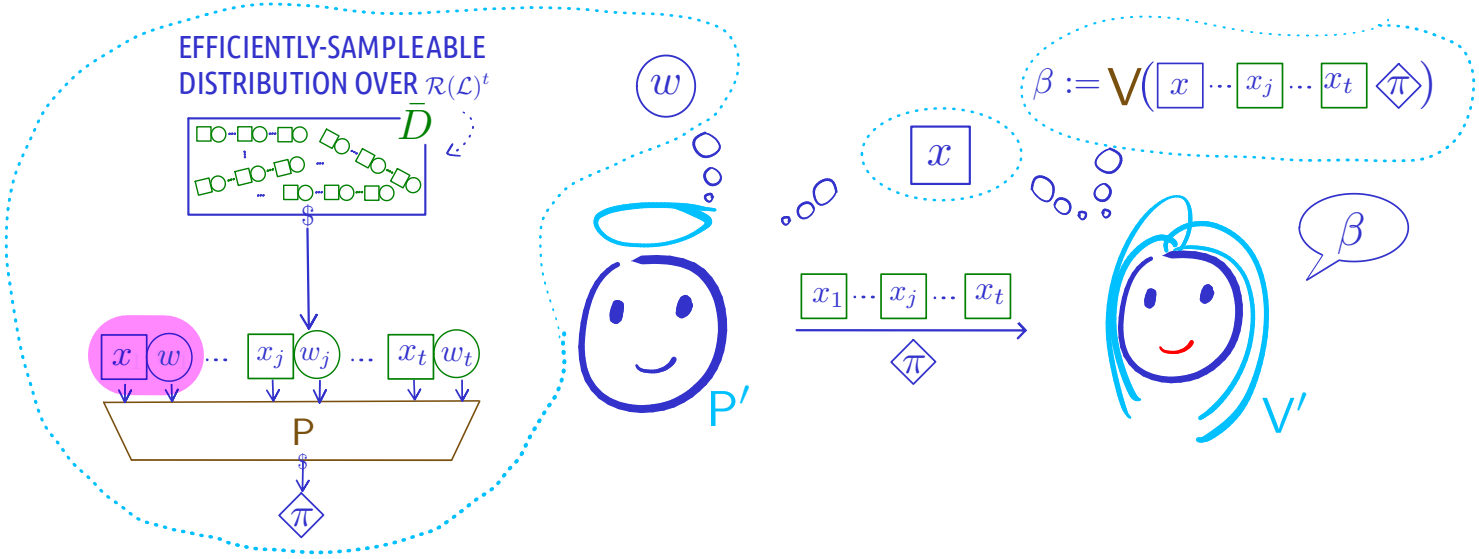

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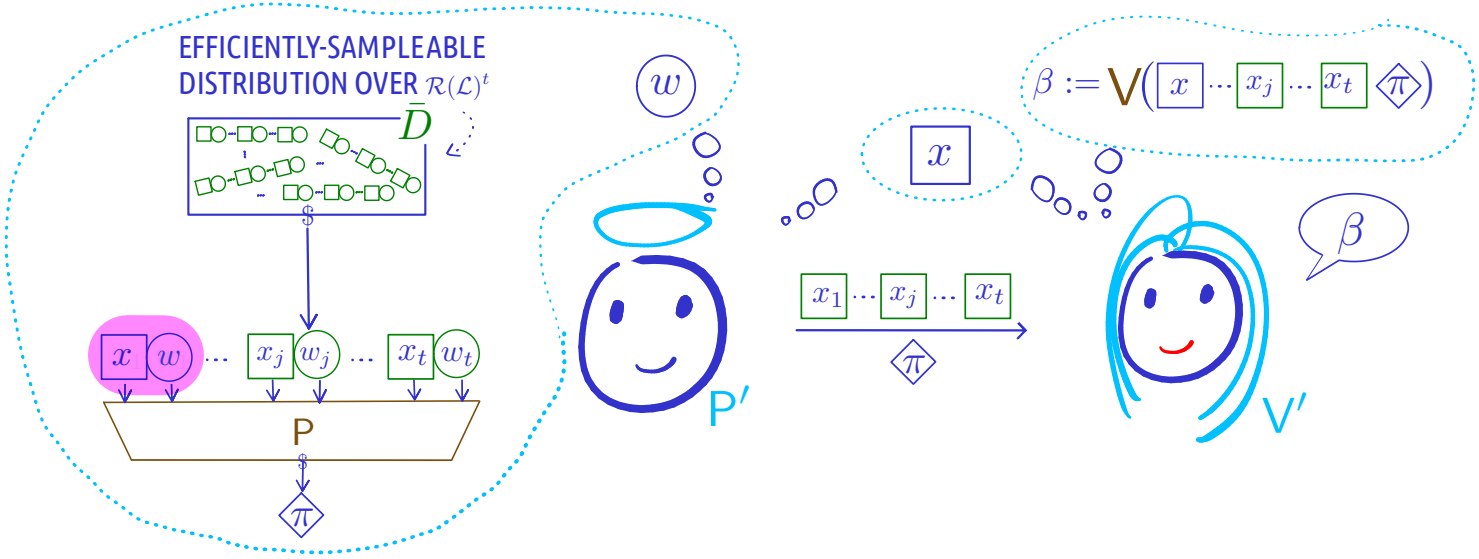

- Π COMPLETE AND SOUND $\Rightarrow$ Π' COMPLETE AND SOUND ✓
- NOT SWI! CONSIDER P THAT DUMPS WITNESS OF 1-ST INPUT ✗
  - COULD BE COMPRESSING, BUT REVEALS WITNESS

BATCH PROTOCOL $\mapsto$ SWI PROTOCOL...

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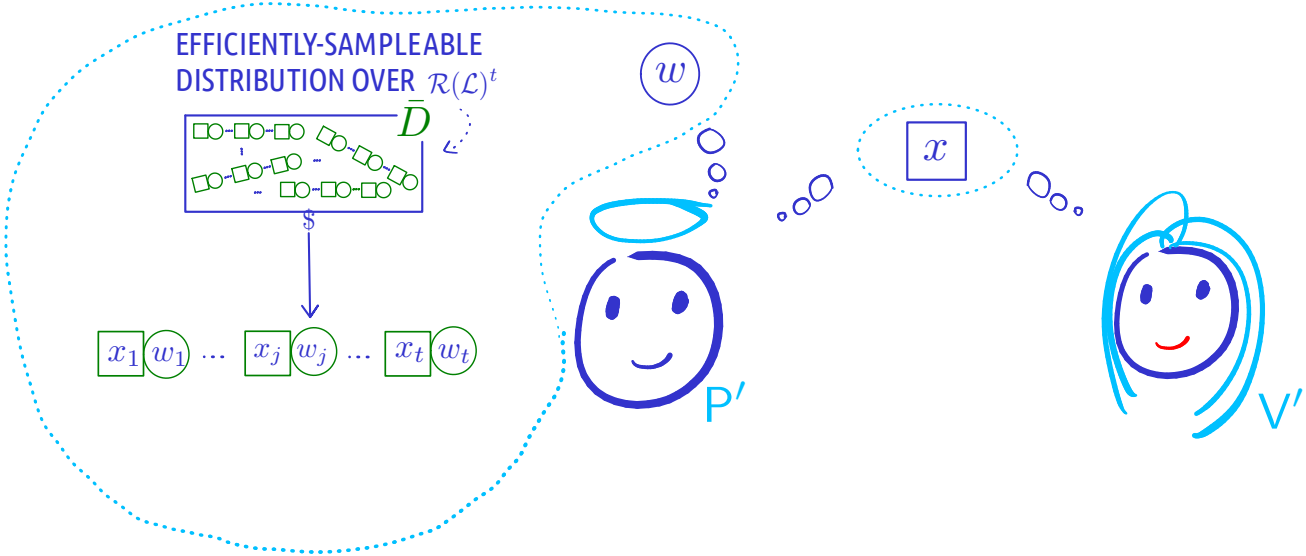

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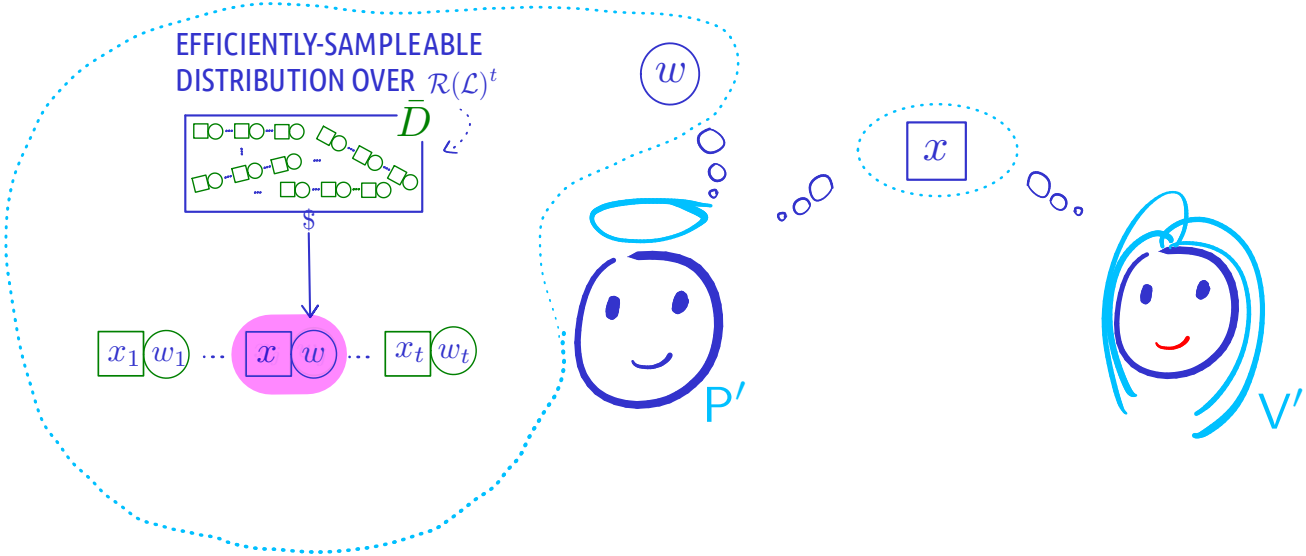

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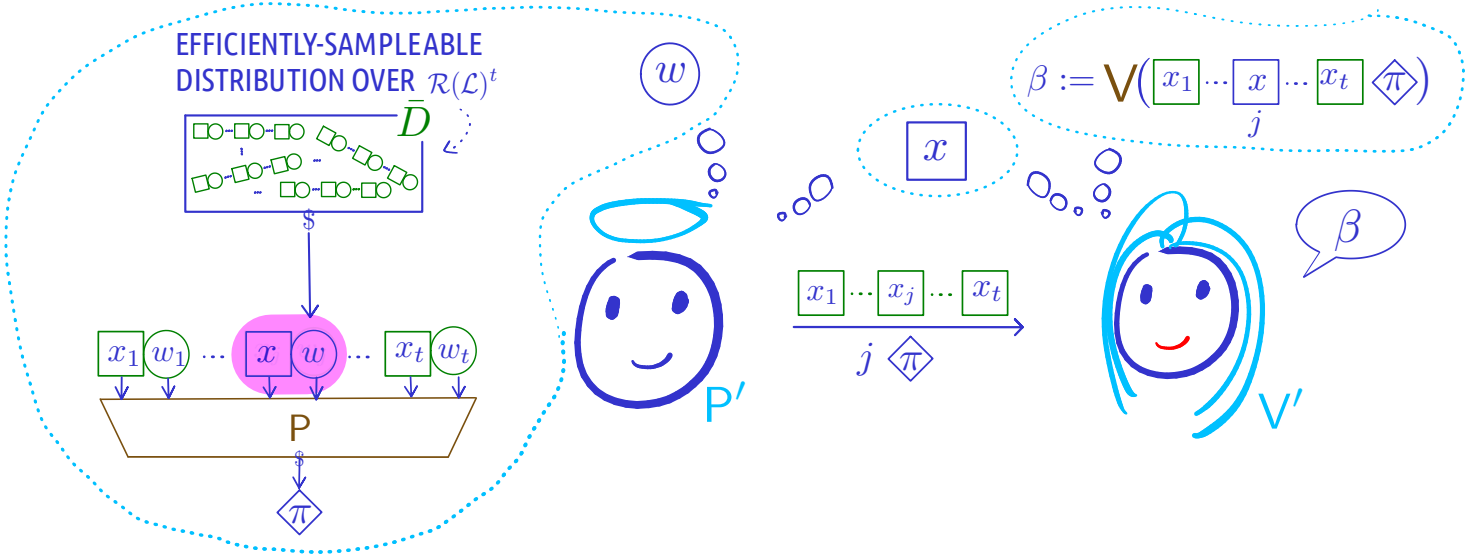

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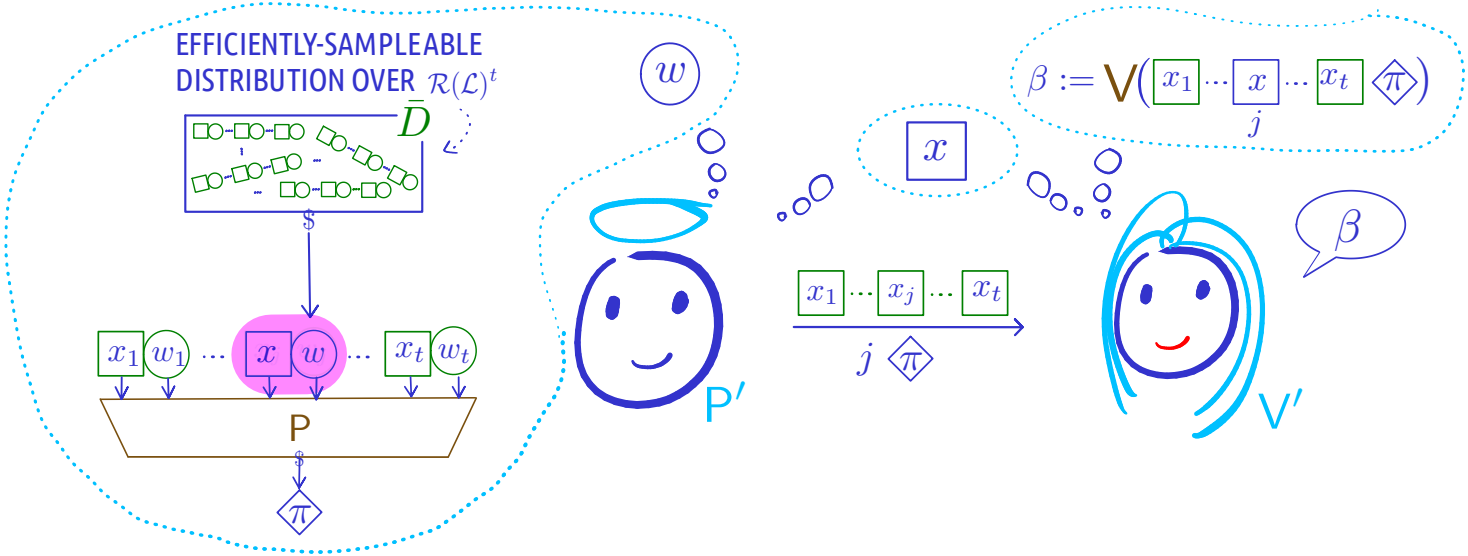

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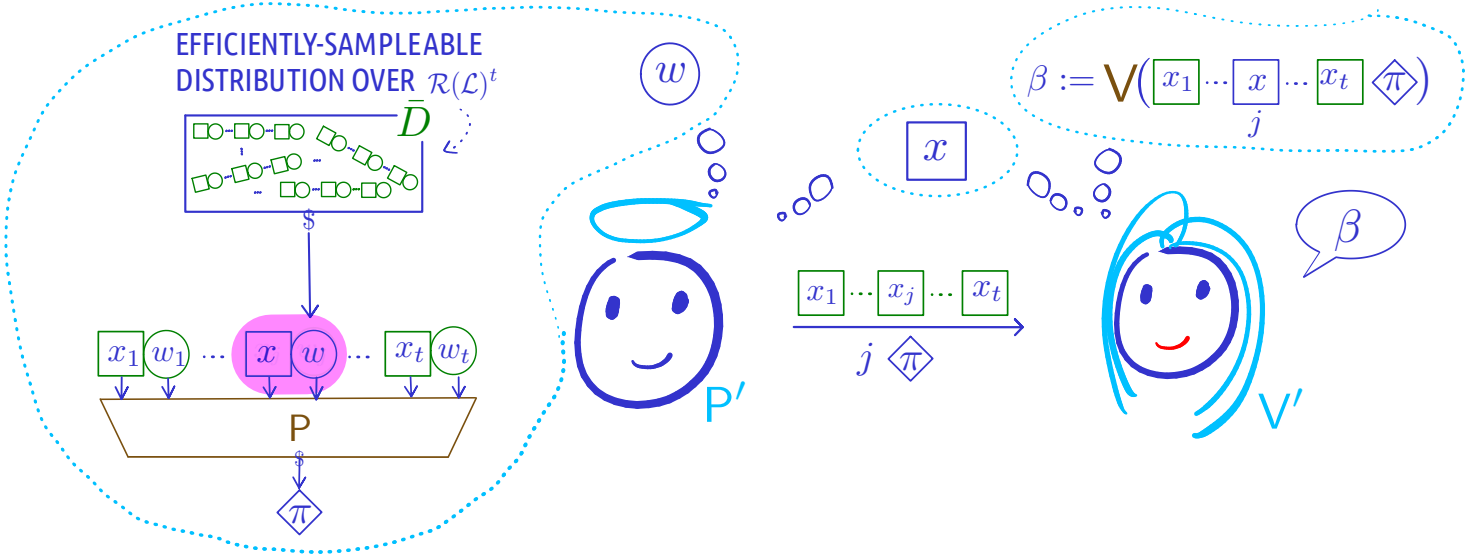

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- SEEMS SWI WHEN P IS COMPRESSING

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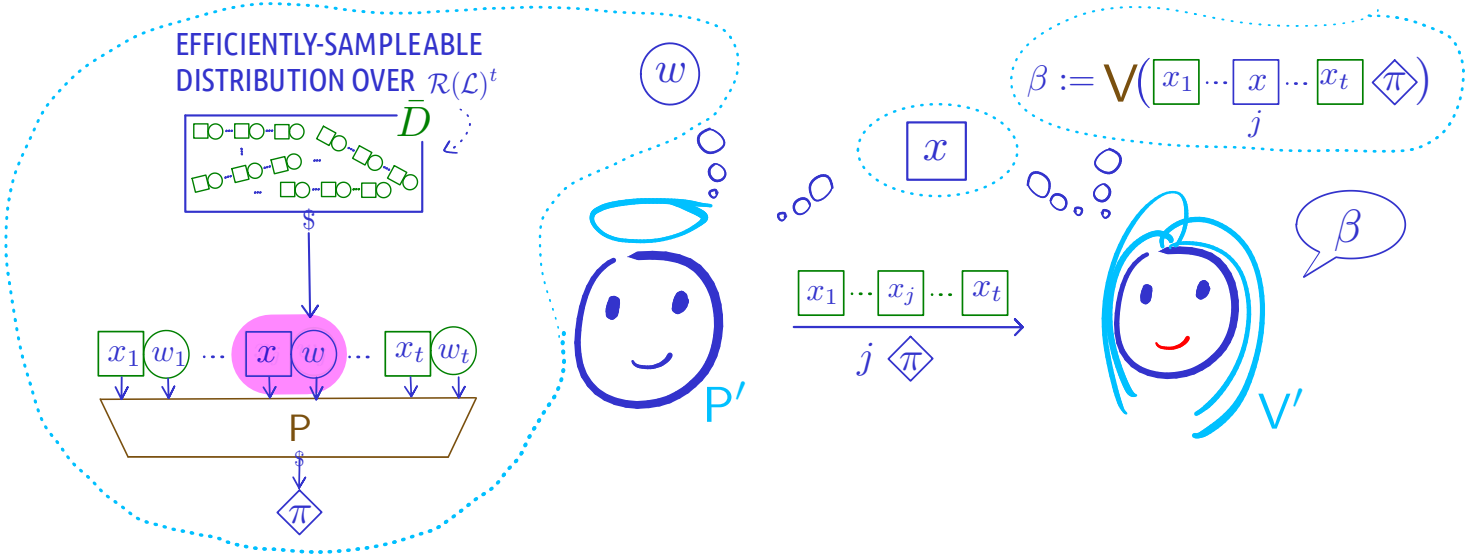

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» HOW TO FIX $\bar{D}$? «

HOW TO FIX $\bar{D}$?

$\forall \mathsf{P} \exists \bar{D} \forall (x, w^0, w^1) : \mathrm{View}_{\mathsf{V}'}(x, w^0) \approx \mathrm{View}_{\mathsf{V}'}(x, w^1)$

$(\boxed{x_1}, \cdots, \boxed{x_j}, \ldots, \boxed{x_t}, j, \langle \pi^0 \rangle) \approx (\boxed{x_1}, \cdots, \boxed{x_j}, \ldots, \boxed{x_t}, j, \langle \pi^1 \rangle)$

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- CHALLENGE: DEALING WITH WORST-CASE WI
  - FIX (x, w^0, w^1) AND CONSIDER PROVER $\mathsf{P}_{\{x\}}$ THAT DUMPS THE WITNESS OF x AS PART OF PROOF π

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- CHALLENGE: DEALING WITH WORST-CASE WI
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- THERE EXISTS $\bar{D}$ THAT WORKS FOR $\mathsf{P}_{\{x\}}$

$x\,w^0\,w^1 \cdots x\,w^0\,w^1 \cdots x\,w^0\,w^1$

HOW TO FIX $\bar{D}$?

$\forall \mathsf{P} \exists \bar{D} \forall (x, w^0, w^1) : \mathrm{View}_{\mathsf{V}'}(x, w^0) \approx \mathrm{View}_{\mathsf{V}'}(x, w^1)$

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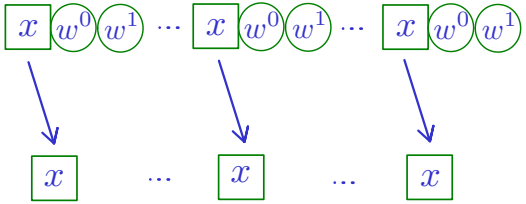

HOW TO FIX $\bar{D}$?

$\forall \mathsf{P} \exists \bar{D} \forall (x, w^0, w^1) : \mathrm{View}_{\mathsf{V}'}(x, w^0) \approx \mathrm{View}_{\mathsf{V}'}(x, w^1)$

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CHALLENGE: DEALING WITH WORST-CASE WI

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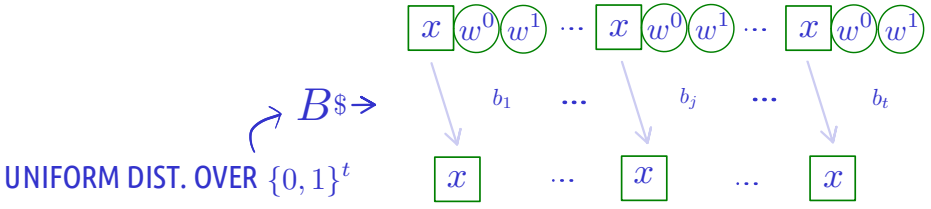

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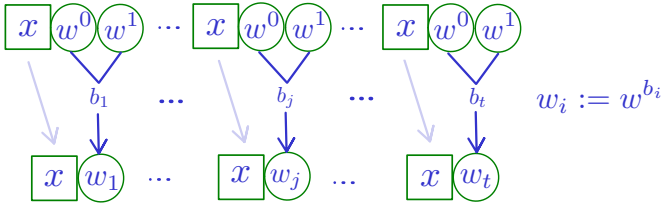
$B^{\$} \rightarrow$ UNIFORM DIST. OVER $\{0,1\}^t$



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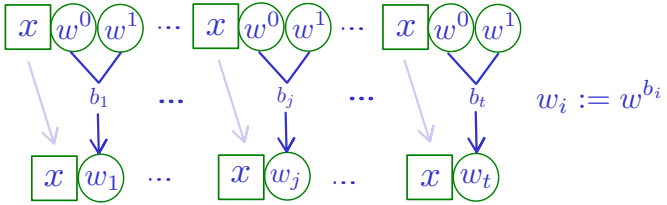

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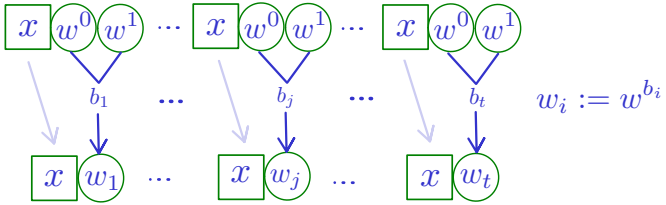

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» HOW TO FIX $\bar{D}$ FOR GENERAL P? «

HOW TO FIX $\bar{D}$?...

[D15,D16]

1)COMPRESSION LEMMA OF DRUCKER/DELL

HOW TO FIX $\bar{D}$?...

[D15,D16]

1)COMPRESSION LEMMA OF DRUCKER/DELL

$$\forall g : \{0,1\}^t \to \{0,1\}^{\rho t} : (j, g(B|_{j\leftarrow 0}) \approx_{\sqrt{\rho}} (j, g(B|_{j\leftarrow 1})$$

UNIFORM OVER $[t]$ UNIFORM OVER $\{0,1\}^t$

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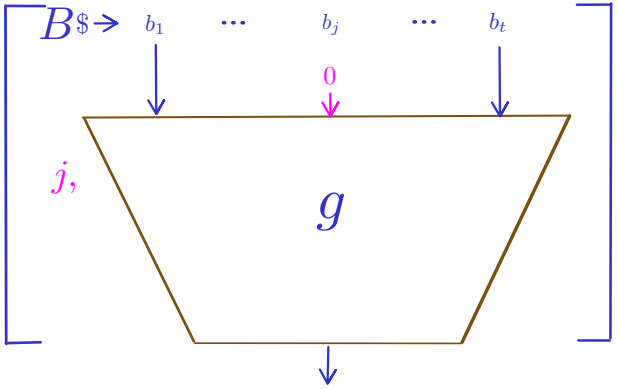

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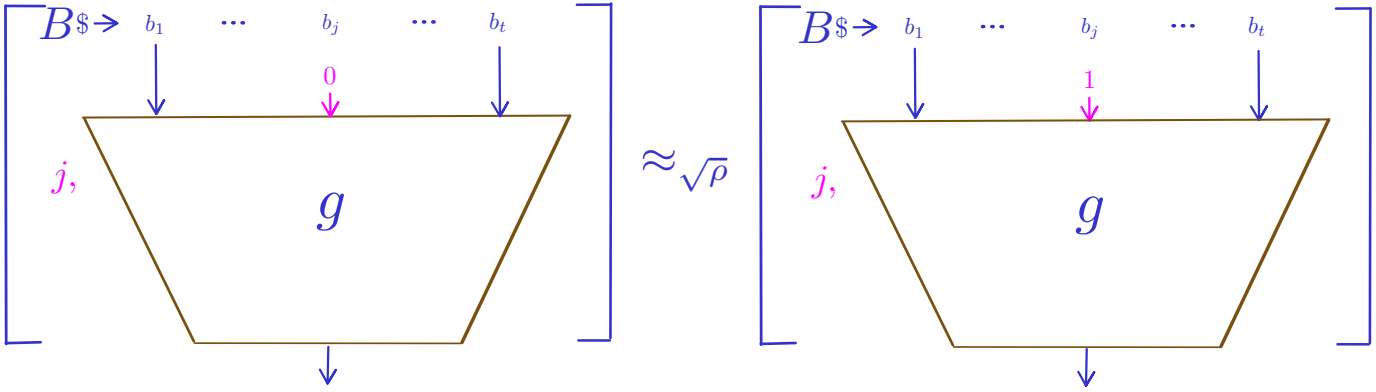

HOW TO FIX $\bar{D}$?...

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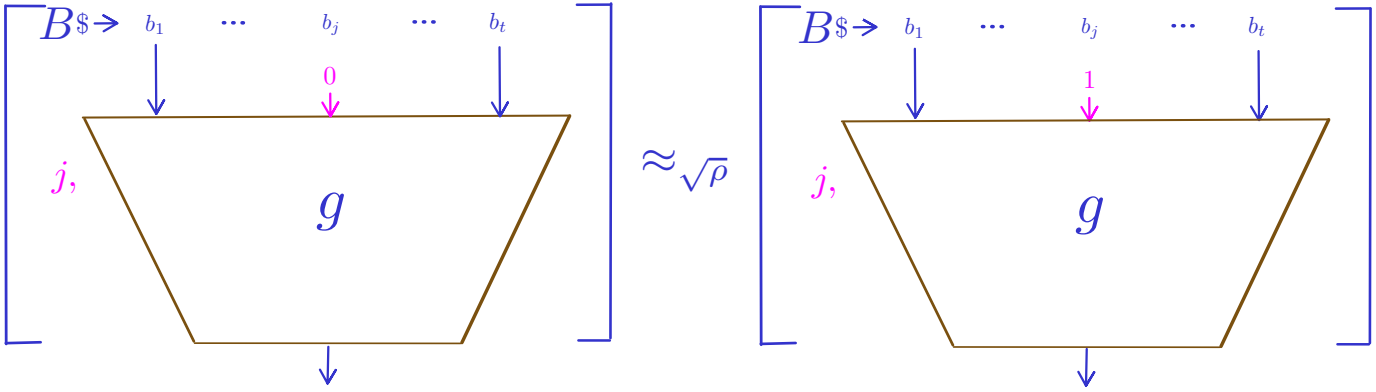

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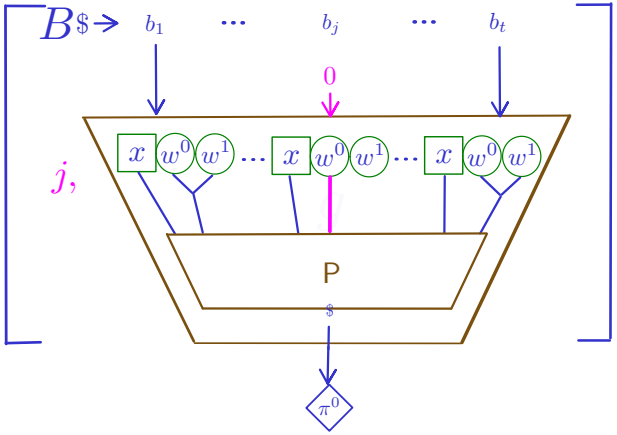

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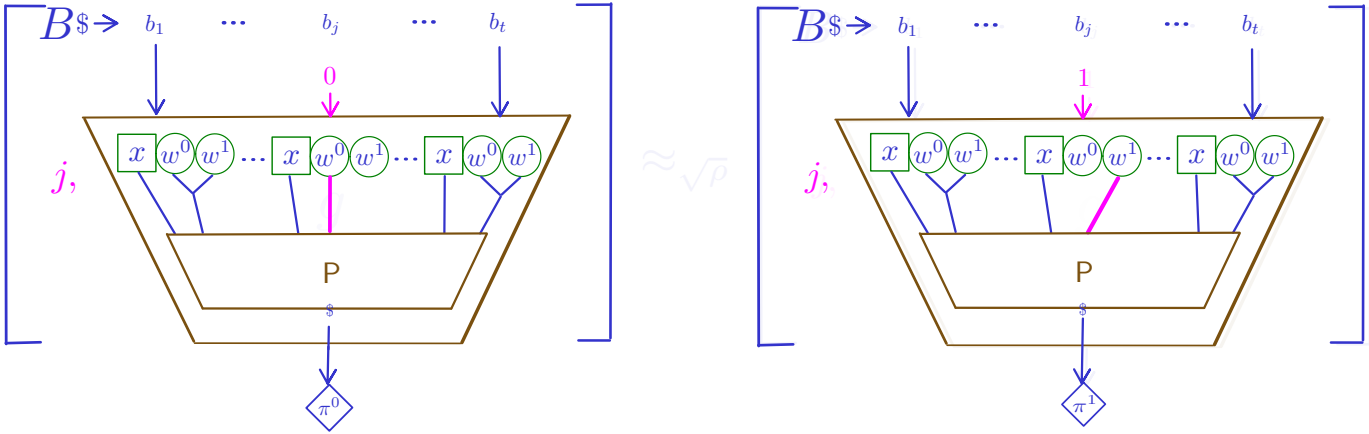

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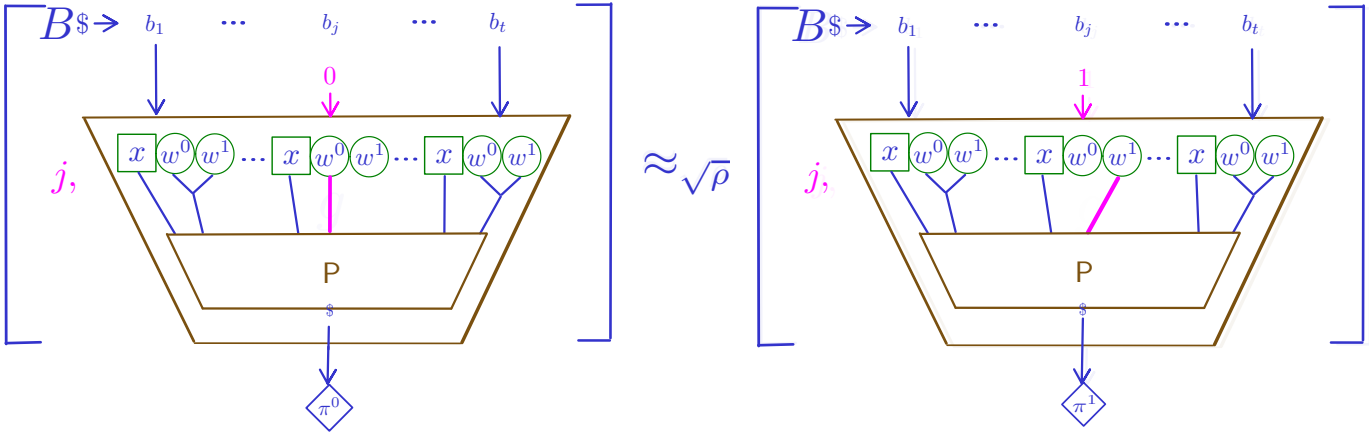

HOW TO FIX $\bar{D}$?...

1*)COMPRESSION LEMMA OF DRUCKER/DELL IMPLIES "DISTRIBUTIONAL WI"

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EFFICIENTLY-SAMPLEABLE
DISTRIBUTION OVER (x, w^0, w^1)

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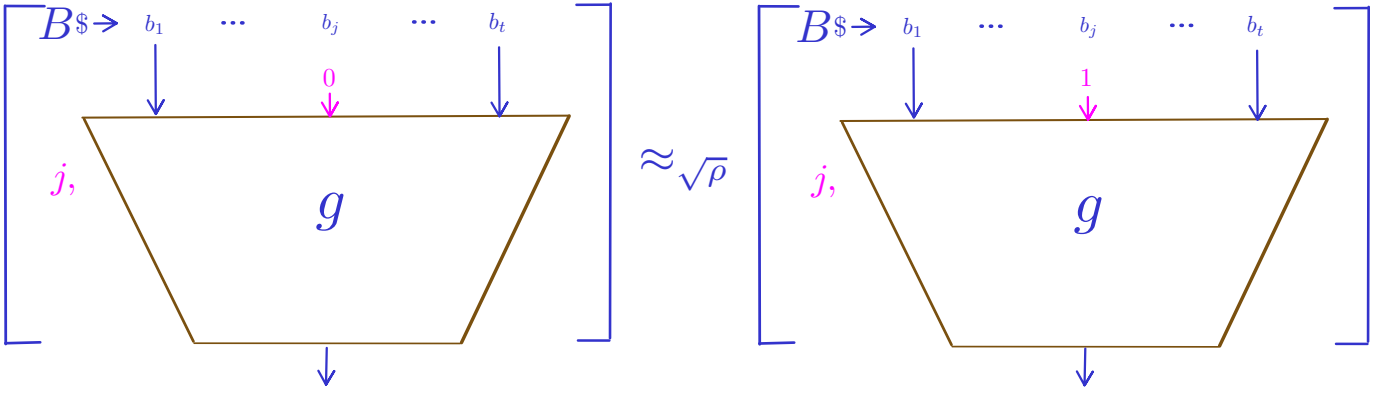

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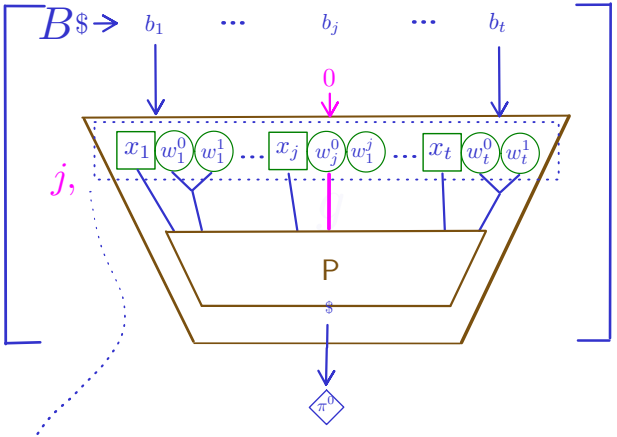
EFFICIENTLY-SAMPLEABLE DISTRIBUTION OVER (x, w^0, w^1)



$\approx_{\sqrt{P}}$

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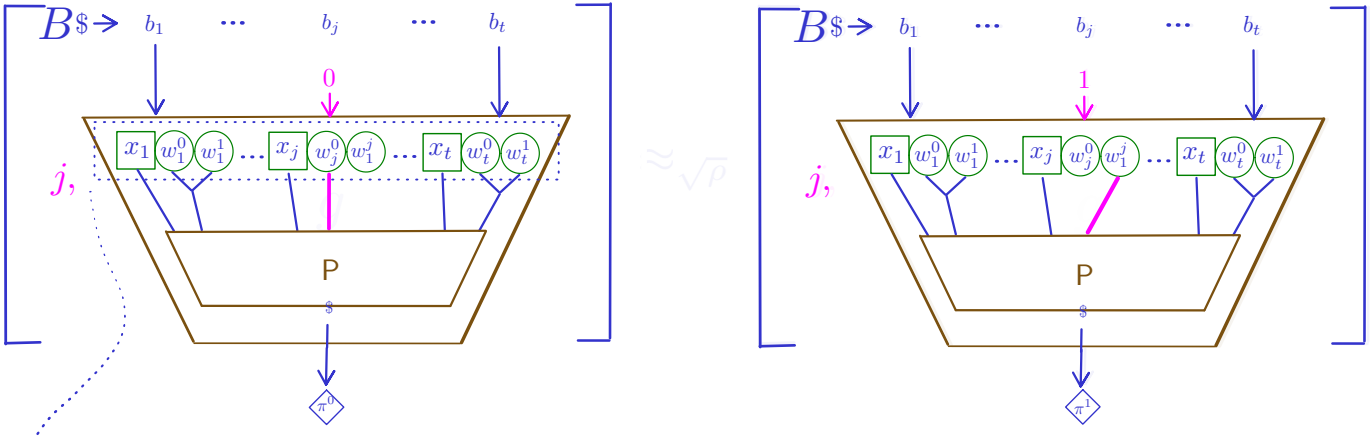

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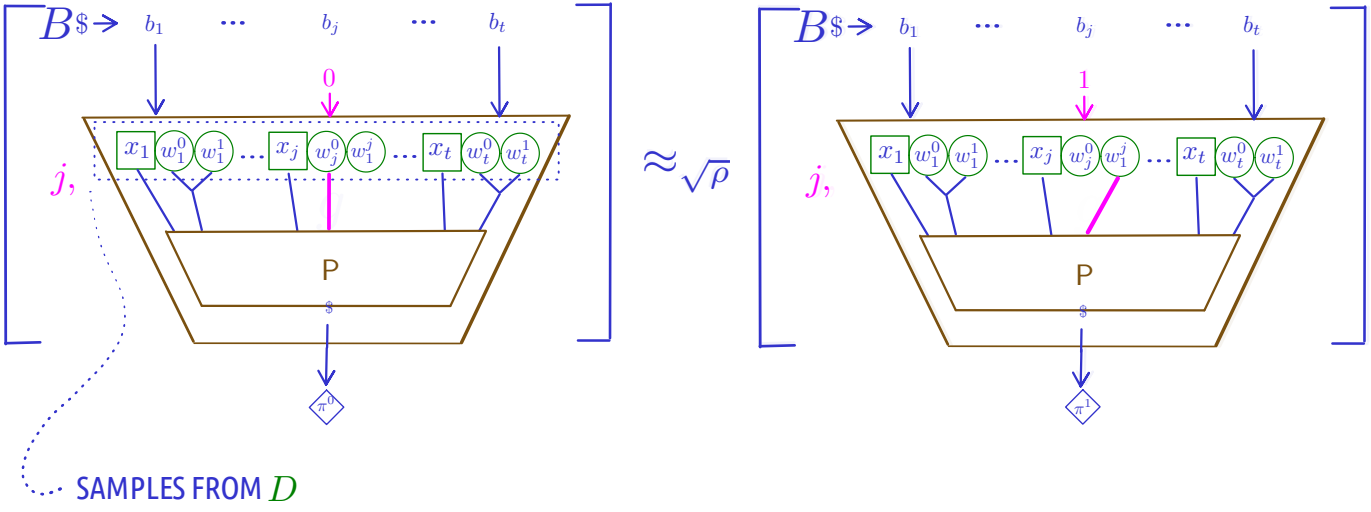

HOW TO FIX $\bar{D}$?...

2)DISTRIBUTIONAL WI TO (WORST-CASE) WI

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2) DISTRIBUTIONAL WI TO (WORST-CASE) WI VIA MINIMAX THEOREM [vN28]

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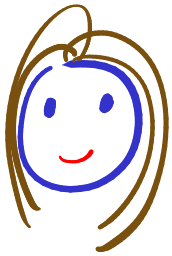
2)DISTRIBUTIONAL WI TO (WORST-CASE) WI VIA MINIMAX THEOREM [vN28]

ZERO-SUM GAME

ROW PLAYER

COLUMN PLAYER



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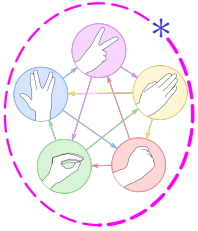

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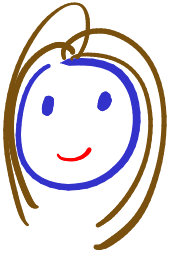

∗ IMAGE: NOJHAN, WIKIPEDIA

HOW TO FIX $\bar{D}$?...

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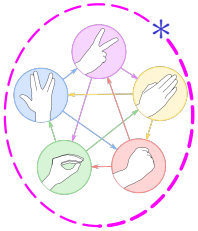
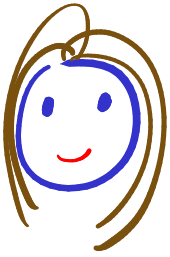


ROW PLAYER

R

COLUMN PLAYER



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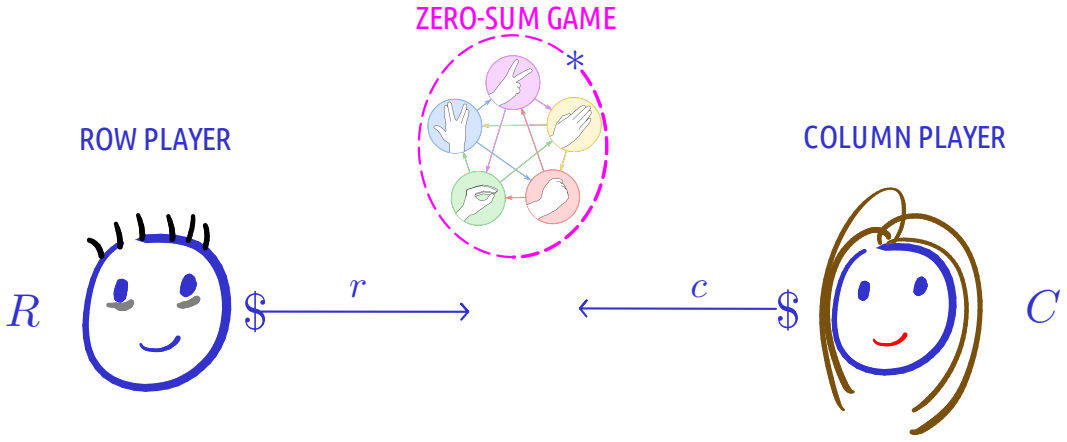

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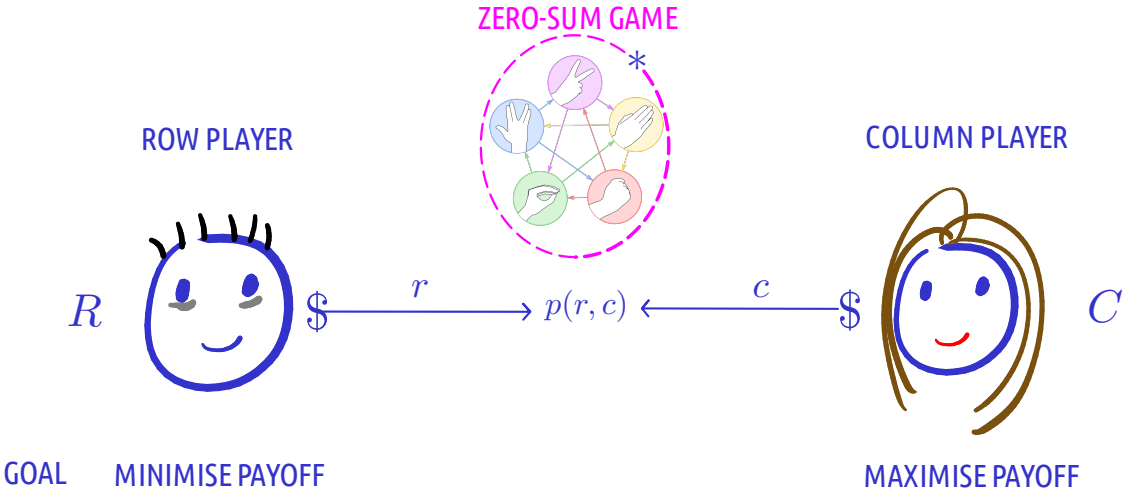

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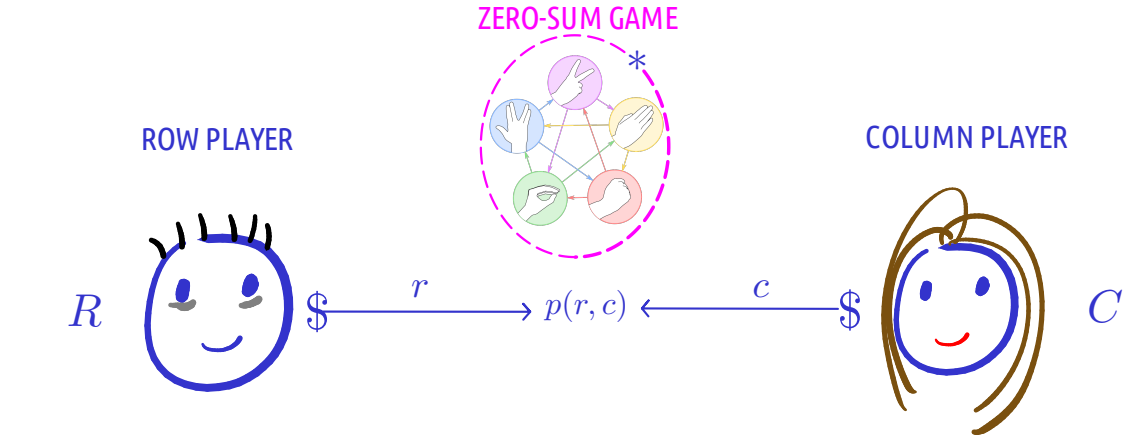

$$\min_{\rho}\max_{\chi}\mathbb{E}_{r\leftarrow\rho,c\leftarrow\chi}[p(r,c)]=\max_{\chi}\min_{\rho}\mathbb{E}_{r\leftarrow\rho,c\leftarrow\chi}[p(r,c)]$$

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HOW TO FIX $\bar{D}$?...

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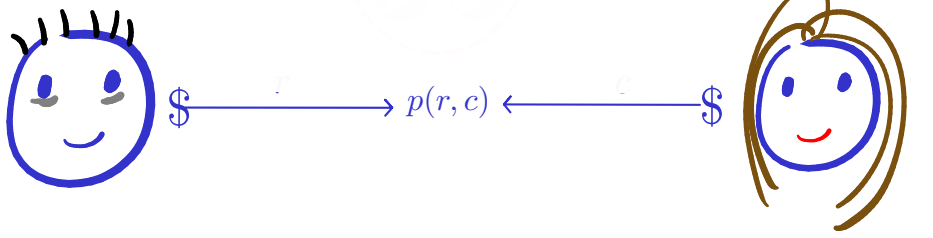
ZERO-SUM GAME



GOAL MINIMISE PAYOFF MAXIMISE PAYOFF

$$\min_{\rho} \max_{\chi} \mathbb{E}_{r \leftarrow \rho, c \leftarrow \chi}[p(r,c)] = \max_{\chi} \min_{\rho} \mathbb{E}_{r \leftarrow \rho, c \leftarrow \chi}[p(r,c)]$$

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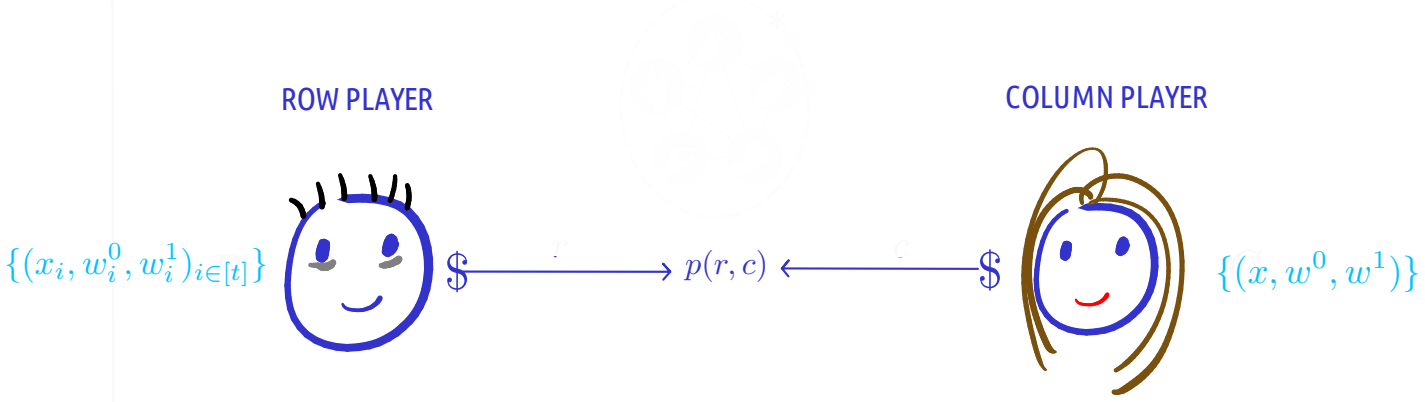

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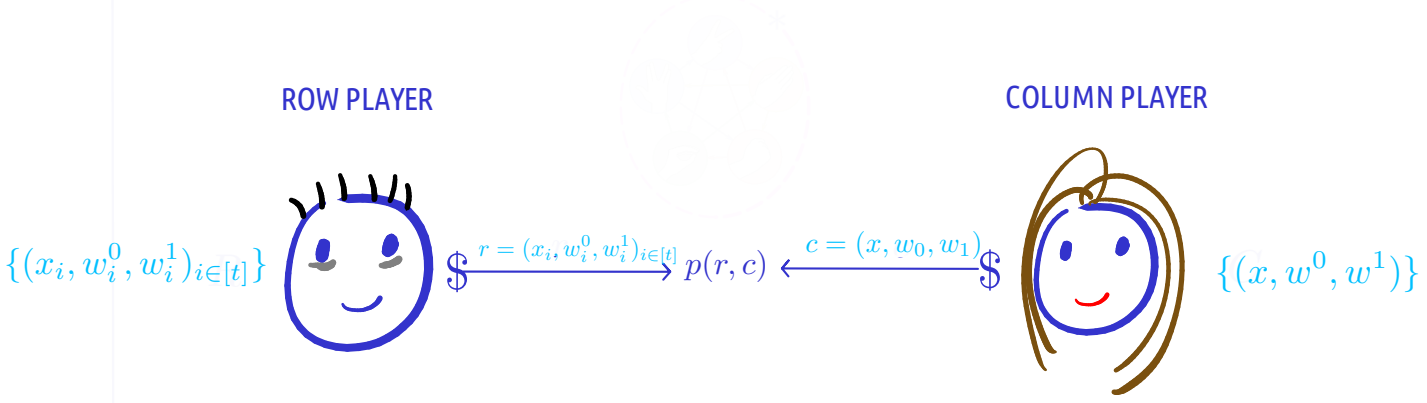
ZERO-SUM GAME



$$\min_{\rho} \max_{\chi} \mathbb{E}_{r\leftarrow\rho, c\leftarrow\chi}[p(r,c)] = \max_{\chi} \min_{\rho} \mathbb{E}_{r\leftarrow\rho, c\leftarrow\chi}[p(r,c)]$$

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* IMAGE: NOJHAN, WIKIPEDIA

HOW TO FIX $\bar{D}$?...

2)DISTRIBUTIONAL WI TO (WORST-CASE) WI VIA MINIMAX THEOREM [vN28]

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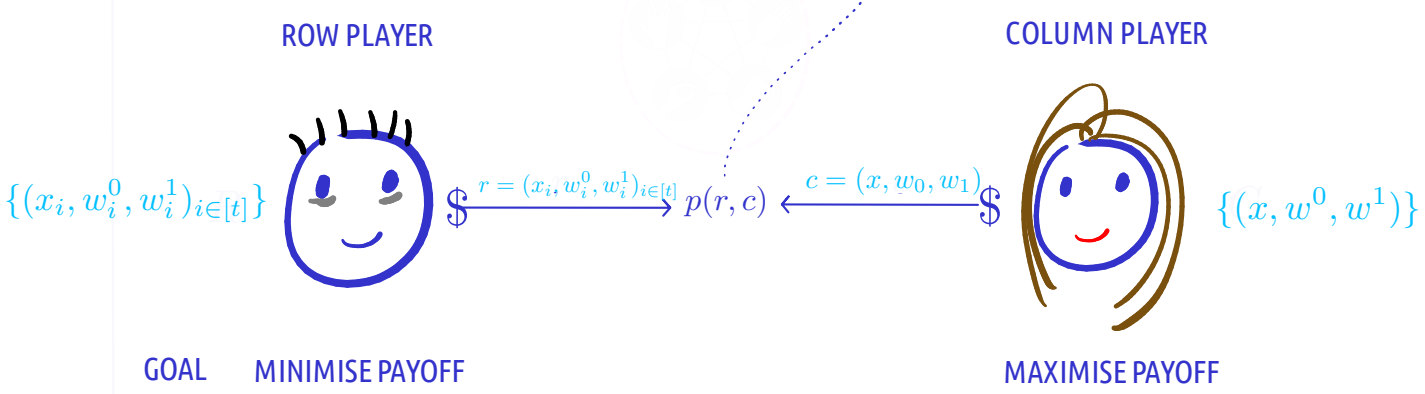

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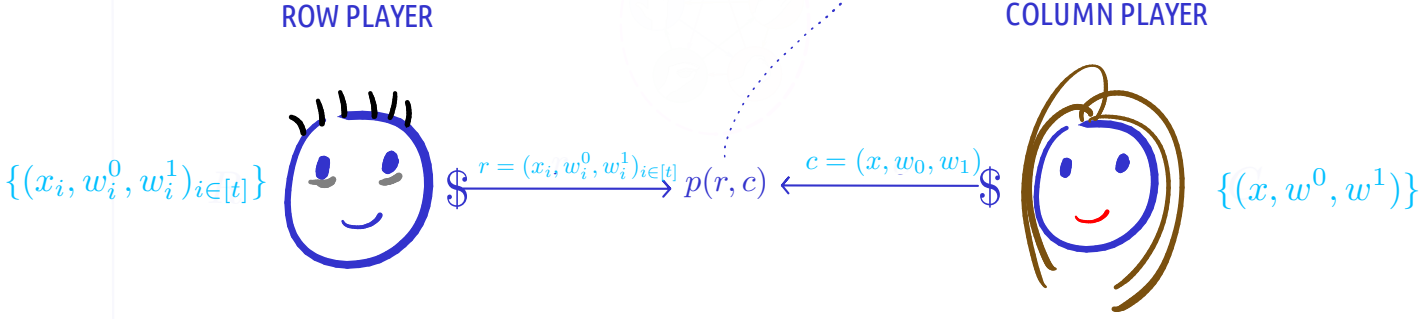

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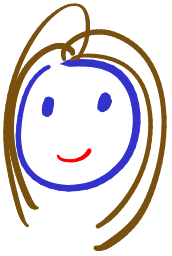
ZERO-SUM GAME

ROW PLAYER

COLUMN PLAYER

$\{(x_i, w_i^0, w_i^1)_{i\in[t]}\}$ $\$$ $\xrightarrow{r=(x_i,w_i^0,w_i^1)_{i\in[t]}}$ $p(r,c)$ $\xleftarrow{c=(x,w_0,w_1)}$ $\$$ $\{(x, w^0, w^1)\}$



GOAL MINIMISE PAYOFF

MAXIMISE PAYOFF

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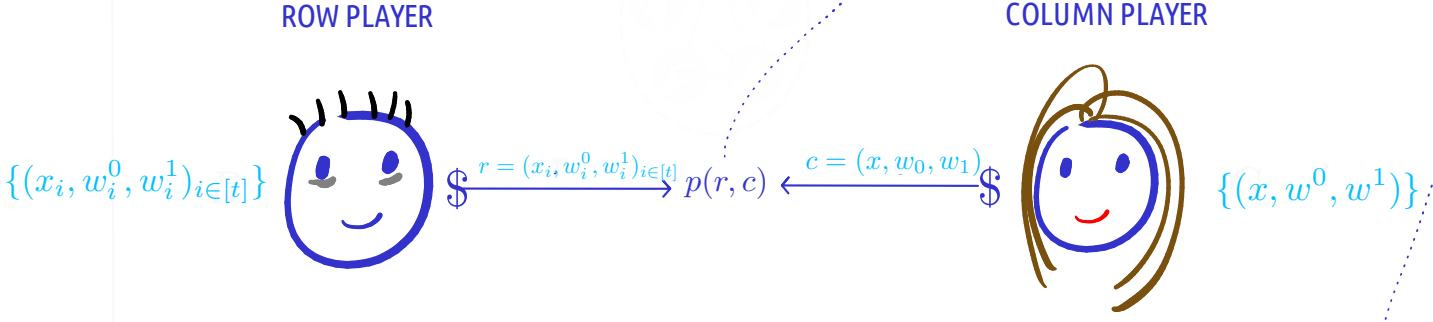

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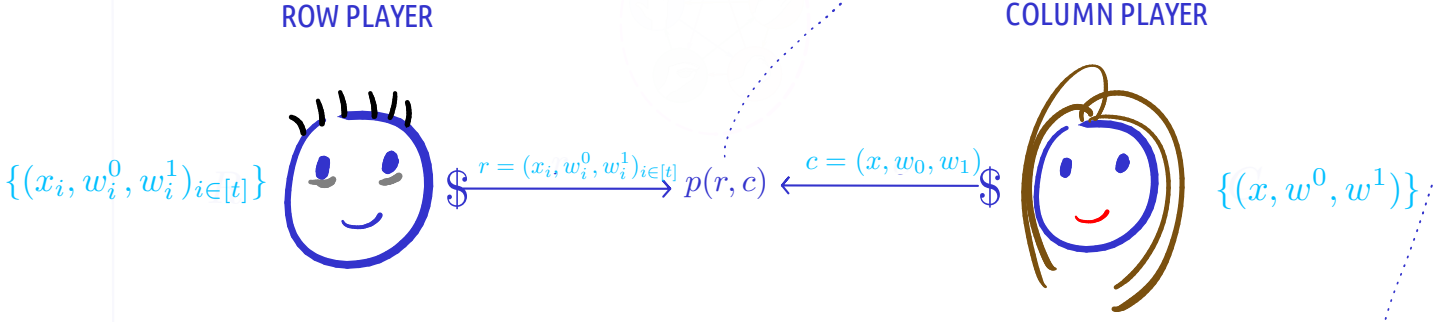

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HOW TO FIX $\bar{D}$?...

SPARSE VERSION OF [LY94]

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ZERO-SUM GAME

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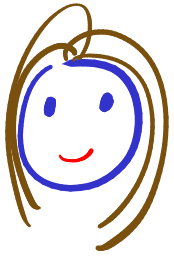

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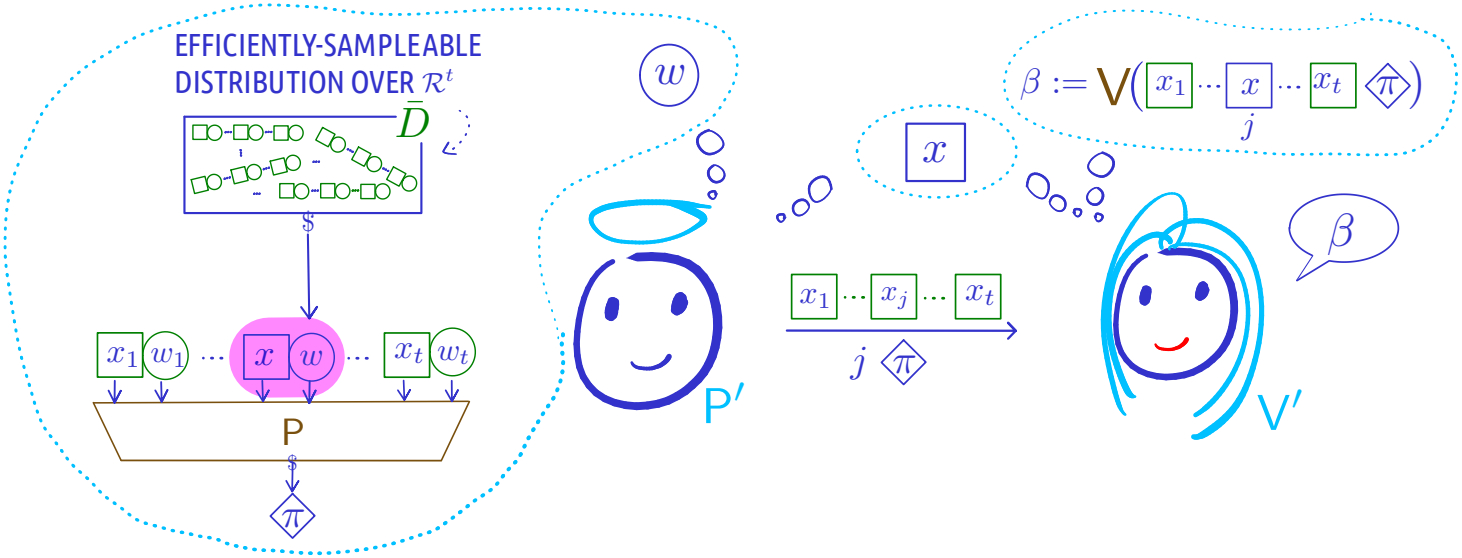
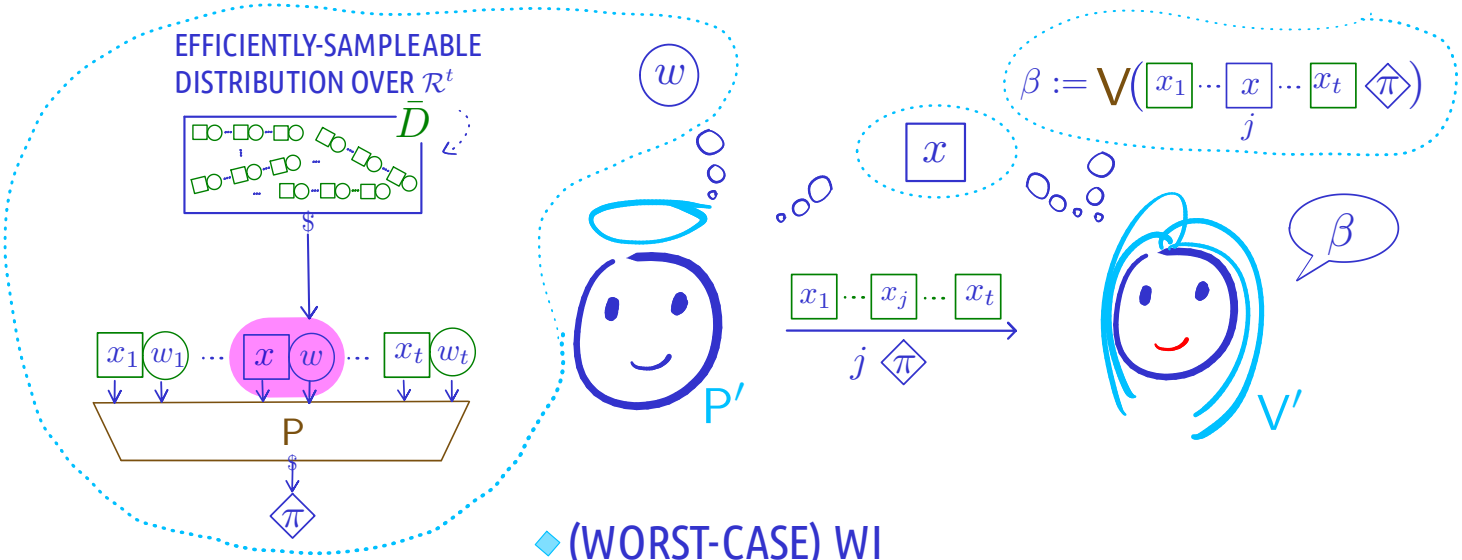
TO RECAP


EFFICIENTLY-SAMPLEABLE
DISTRIBUTION OVER $\mathcal{R}^t$
$\bar{D}$
x_1 w_1 ⋯ x w ⋯ x_t w_t
P
π
w
P′
x
x_1 ⋯ x_j ⋯ x_t
j π
$\beta := \mathsf{V}(x_1 \cdots \underset{j}{x} \cdots x_t\ \pi)$
β
V′

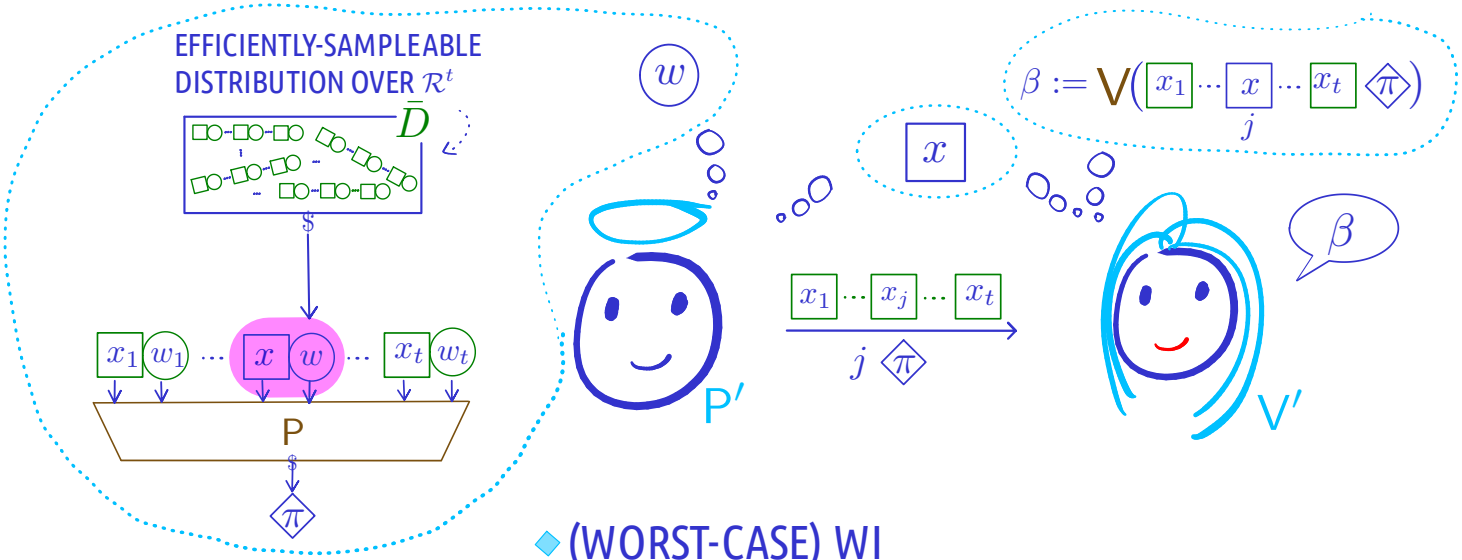
TO RECAP



- (WORST-CASE) WI

$\forall \mathsf{P} \exists \bar{D} \forall D$: Views indistinguishable

TO RECAP



◆ (WORST-CASE) WI

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↓

REDUCES TO DISTRIBUTIONAL WI VIA (SPARSE) MINIMAX THEOREM

$\forall \mathsf{P} \forall D \exists \bar{D}$: Views indistinguishable

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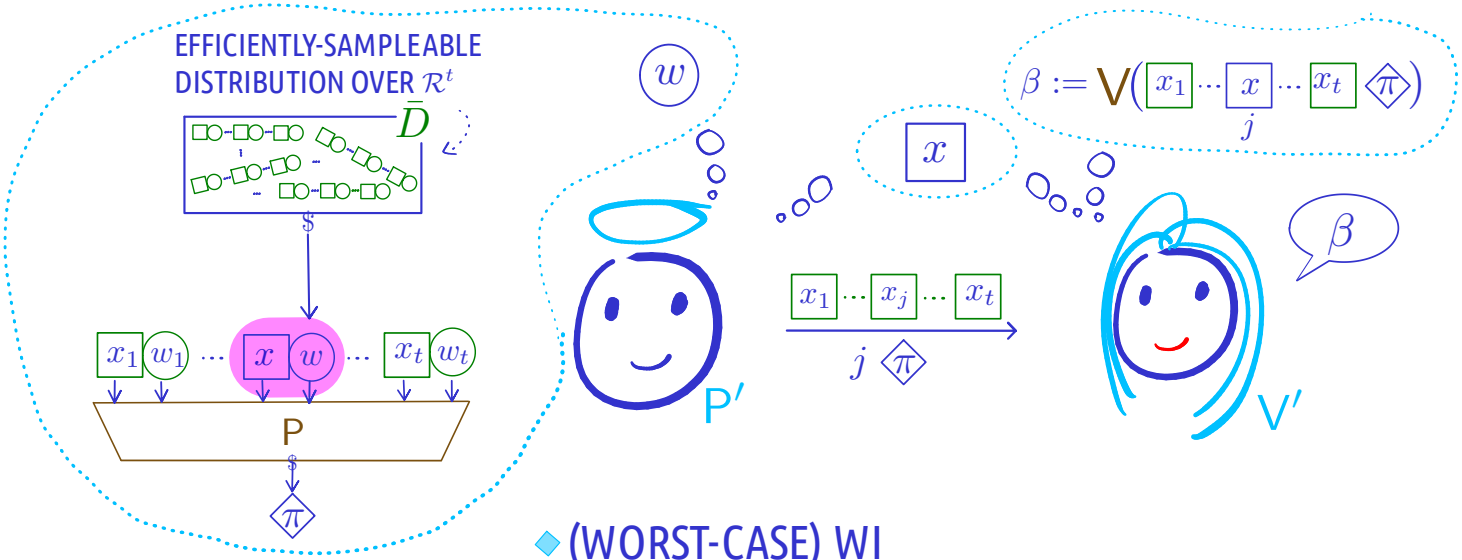

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↓

DISTRIBUTIONAL WI VIA COMPRESSION LEMMA OF DRUCKER/DELL

PLAN

- BACKGROUND
  - DEFINITIONS: BATCH PROOFS STAT. WITNESS INDISTINGUISHABILITY (SWI)
  - WHAT CAN BE BATCHED?
- MAIN RESULT
  - BATCH PROOFS $\Rightarrow$ SWI PROOFS
  - TECHNICAL OVERVIEW
- **MORE RESULTS**
  - ONE-WAY FUNCTION + BATCH ARGUMENTS $\Rightarrow$ SZK ARGUMENTS
  - NON-INTERACTIVE (NI) BATCH ARGUMENTS $\Rightarrow$ NI SZK ARGUMENTS
- OPEN QUESTIONS

BATCH ARGUMENTS $\Rightarrow$ SWI ARGUMENTS

INTERACTIVE SETTING

NON-INTERACTIVE (NI) SETTING (CRS MODEL)

BATCH ARGUMENTS $\Rightarrow$ SWI ARGUMENTS

INTERACTIVE SETTING

BATCH ARGUMENTS

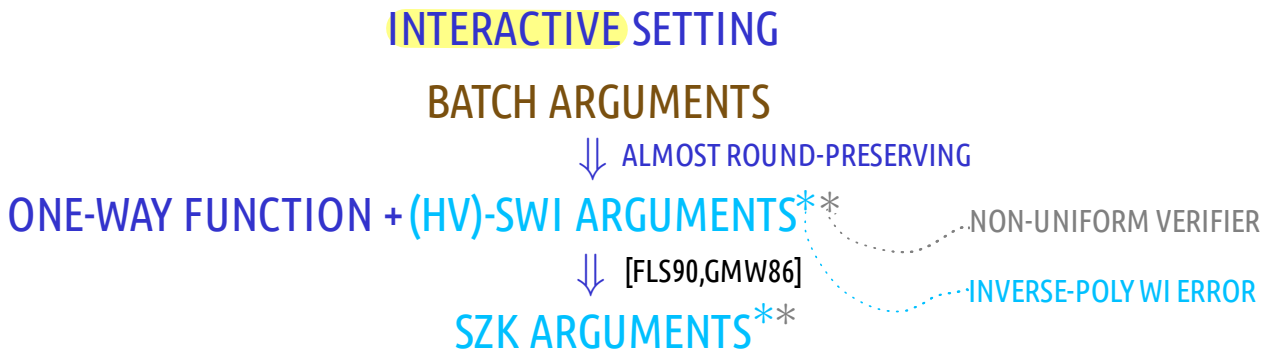
$\Downarrow$ ALMOST ROUND-PRESERVING

(HV)-SWI ARGUMENTS**

- NON-UNIFORM VERIFIER
- INVERSE-POLY WI ERROR

NON-INTERACTIVE (NI) SETTING (CRS MODEL)

BATCH ARGUMENTS $\Rightarrow$ SWI ARGUMENTS



BATCH ARGUMENTS $\Rightarrow$ SWI ARGUMENTS

INTERACTIVE SETTING

BATCH ARGUMENTS

$\Downarrow$ ALMOST ROUND-PRESERVING

ONE-WAY FUNCTION + (HV)-SWI ARGUMENTS** — NON-UNIFORM VERIFIER

$\Downarrow$ [FLS90,GMW86] — INVERSE-POLY WI ERROR

COLLISION RESISTANCE $\Rightarrow$ SZK ARGUMENTS**

NON-INTERACTIVE (NI) SETTING (CRS MODEL)

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NON-INTERACTIVE (NI) SETTING (CRS MODEL)

(SOMEWHERE-EXTRACTABLE) NI BATCH ARGUMENTS

$\Downarrow$

NISWI ARGUMENTS**

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◆ BATCH ARGUMENTS ⇒ SWI ARGUMENTS

INTERACTIVE SETTING

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NON-INTERACTIVE (NI) SETTING (CRS MODEL)

(SOMEWHERE-EXTRACTABLE) NI BATCH ARGUMENTS

⇓

NISWI ARGUMENTS**

⇓ [FLS90]

LOSSY PKE + NISZK ARGUMENTS**

⇓ PRIVACY AMPLIFICATION [GJS19,LM20]

NISZK ARGUMENTS*

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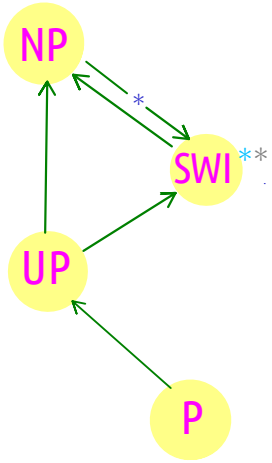

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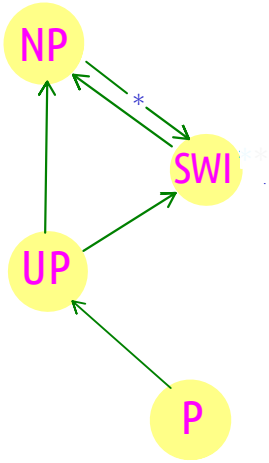

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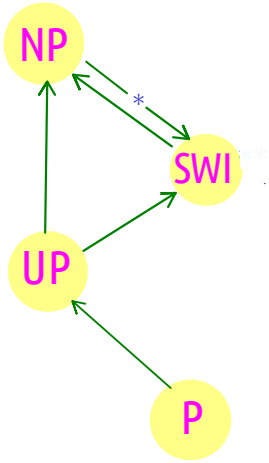

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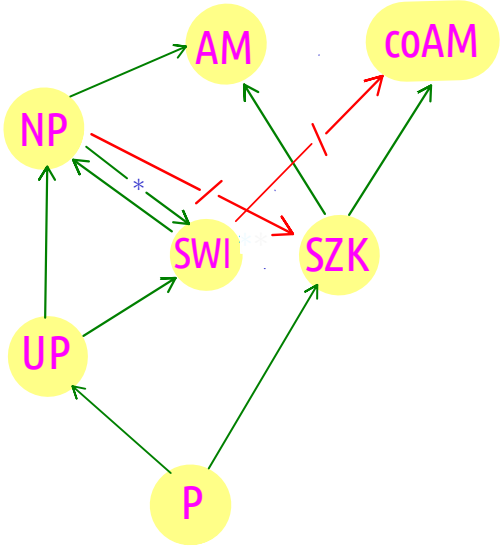

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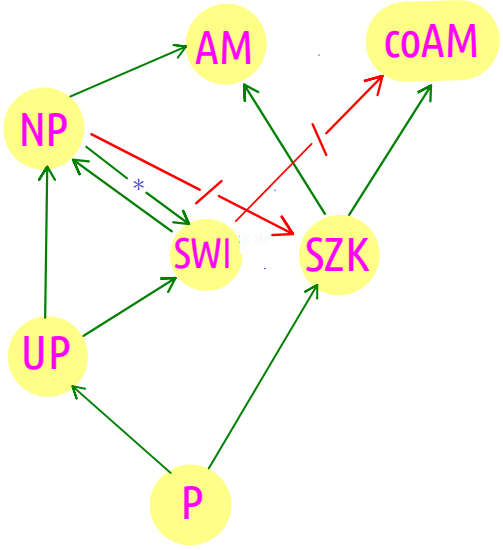

THANK YOU!

NATIONAL
RESEARCH
FOUNDATION
PRIME MINISTER'S OFFICE
SINGAPORE

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